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NAVAL POSTGRADUATE SCHOOL

Monterey, California



A STUDY OF THE PROPERTIES OF A NEW GOODNESS-OF-FIT TEST

by

Richard Franke

and

Toke Jayachandran

Technical Report for Period
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Prepared for: Chief of Naval Research
Arlington, Virginia 22217

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ABSTRACT

We investigate the power properties of a new goodness-of-fit test proposed by Foutz (1980). This new test is compared with the Chi squared test and the Kolmogorov-Smirnov (K-S) test for normality when the samples come from (i) the family of asymmetric stable distributions, (ii) mixtures of normal distributions, and (iii) the Pearson family. The general conclusion is that the new test performs better than the Chi squared and the K-S test when the parent distribution is heavy-tailed. If the hypothesized distribution differs from the true distribution in location only, the new test does not do as well as the other two.

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1. INTRODUCTION

In a recent article Foutz (1980) introduced a new test for goodness-of-fit, to be called the F_n test in the sequel. Although the test was proposed for fitting a continuous p-variate distribution, it applies equally well to univariate problems. The null distribution of the test statistic was shown to be distribution free as well as being independent of the number of variates p. Foutz obtained an integral representation for the null CDF of F_n ; explicit expressions for this CDF were given for sample sizes 2 and 3. Closed form solutions for the CDF for larger sample sizes are quite hard to derive and Foutz has provided a large sample normal approximation to the null distribution of F_n .

In a preliminary comparison with ten replications of 50 simulated samples from (i) a mixture of uniform distributions and (ii) a standard normal distribution, Foutz found that the F_n test outperformed both the Chi squared test and the Kolmogorov-Smirnov (K-S) test.

In this paper we present the results of an extensive investigation to compare the three goodness-of-fit tests, the Chi squared test, the K-S test and the F_n test. Members from three families of distributions, viz., the family of asymmetric stable distributions, mixtures of normals, and the Pearson family have been selected to represent the true underlying distribution of the samples. The goodness-of-fit tests are applied to test the hypothesis that the samples are from a standard normal distribution. The measure of comparison used in the study is the empirical power, based on 5000 replications of each of the tests.

The Chi squared and Kolmogorov-Smirnov statistics were computed using the IMSL⁺ routines GFIT and NKS1. The number of cells used in the Chi

⁺International Mathematical and Statistical Libraries, 7500 Bellaire Boulevard, Houston, TX 77036

squared computation was 6, 8, and 8 for sample sizes of 20, 30, and 50, respectively.

A brief discussion of the F_n test is given in Section 2 and a description of the simulation is in Section 3. The results of the simulation are presented in Section 4. FORTRAN codes used for the simulation and detailed tables of simulation results are in Appendices I and II.

2. F_n TEST

The F_n test is based on a comparison of a continuous empirical distribution function (CEDF) with the hypothesized CDF. The CEDF is obtained by "spreading" the total mass over "statistically equivalent blocks" generated by the sample. As shown in Anderson (1966) and Foutz (1980), given the order statistics of a random sample of size $(n-1)$, n statistically equivalent blocks that partition the sample space can be constructed in many different ways by choosing what are called cutting functions. An intuitively appealing set of blocks, which is the one used in this study is obtained by choosing the identity functions for the cutting functions. In this case, the n statistically equivalent blocks are $B_1 = (-\infty, x_{(1)}]$, $B_2 = (x_{(1)}, x_{(2)}]$, ..., $B_n = (x_{(n-1)}, \infty)$ where $x_{(1)}, x_{(2)}, \dots, x_{(n-1)}$ are the order statistics of a sample of size $n-1$. The CEDF is constructed by spreading a mass $\frac{1}{n}$ continuously and in the same proportion as the hypothesized CDF over each block. If H_o is the hypothesized CDF and \hat{H}_n the CEDF, the test statistic F_n is defined as

$$F_n = \sup_x |H_n(x) - H_o(x)| \quad (1)$$

Let D_i , $i = 1, 2, \dots, n$ be the probability contents of the blocks B_i under the null CDF H_o , i.e., $D_i = P[x \in B_i | H_o]$. A computationally convenient form for F_n can be shown to be

$$F_n = \sum_{i=1}^n \max(0, \frac{1}{n} - D_i) . \quad (2)$$

Foutz has provided the null distribution of F_n in integral form and derived closed form solutions for $n = 3, 4$. Simplifying the expressions given by Foutz for $n = 3, 4$ yields

$$P(F_3 \leq x) = \begin{cases} 6x^2 & 0 \leq x \leq \frac{1}{3} \\ 1 - 3\left(\frac{1}{2} - x\right)^2 & \frac{1}{3} < x \leq 2/3 \\ 1 & x > 2/3 \end{cases} \quad (3)$$

and

$$P(F_4 \leq x) = \begin{cases} 20x^3 & 0 \leq x \leq \frac{1}{4} \\ -20x^3 + 18x^2 - \frac{9}{4}x + \frac{1}{16} & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 1 - 4\left(\frac{3}{4} - x\right)^3 & \frac{1}{2} \leq x \leq \frac{3}{4} \\ 1 & x > \frac{3}{4} \end{cases} \quad (4)$$

We have also obtained the expression for the null CDF for $n = 5$,

$$P(F_5 \leq x) = \begin{cases} 70x^4 & 0 \leq x \leq \frac{1}{5} \\ -105x^4 + 80x^3 - 12x^2 + \frac{16}{25}x - \frac{1}{125} & \frac{1}{5} < x \leq \frac{2}{5} \\ 45x^4 - 80x^3 + \frac{228}{5}x^2 - \frac{176}{25} + \frac{31}{125} & \frac{2}{5} < x \leq \frac{3}{5} \\ 1 - 5\left(\frac{4}{5} - x\right)^4 & \frac{3}{5} < x \leq \frac{4}{5} \\ 1 & x > \frac{4}{5} \end{cases} \quad (5)$$

As is evident the exact distribution is quite difficult to obtain for higher sample sizes. A large sample normal approximation, due to Foutz is given by

$$\lim_{n \rightarrow \infty} P\left[F_n \leq x\right] = \Phi\left[\frac{n(x - e^{-1})}{(2e^{-1} - 5e^{-2})^{1/2}}\right] \quad (6)$$

where Φ is the standard normal CDF. For $n-1 = 20, 30, 50$ we used this approximation to test the hypothesis that a simulated sample from $U[0,1]$, a uniform distribution on $[0,1]$, is in fact from that distribution. In 80,000 replications, the observed significance level (number of hypothesis rejections/80,000) was consistently smaller than the nominal value as can be seen from

TABLE 1

EMPIRICAL SIGNIFICANCE LEVEL OF
FOUTZ F_n TEST USING ASYMPTOTIC APPROXIMATION
(80,000 REPLICATIONS)

Sample Size	20	30	50
Significance Level			
.10	.0757	.0800	.0859
.05	.0372	.0399	.0428
.01	.0082	.0083	.0093

Table 1. We therefore constructed a Monte Carlo CDF of F_n for $n-1 = 2, 3, 4, 20, 30, 50$ based on 25,000 computer generated F_n values; these represent values of the F_n statistic for testing the hypothesis that a set of samples from $U[0,1]$ is in fact from $U[0,1]$. A comparison of the Monte Carlo CDF with the exact CDF for $n-1 = 2, 3, 4$ is provided in Table 2. It can be seen that the Monte Carlo CDF provides a reasonable approximation even for small n .

The power properties of the F_n test detailed in this paper are based on the Monte Carlo CDF of F_n . Critical values obtained from the Monte Carlo simulation for significance levels .01, .05, .1 and $n-1 = 20, 30, 50$ are in Table 3. In Table 4 we present the observed significance level in 225,000 replications when testing if a set of samples from $U[0,1]$ is in fact from $U[0,1]$.

TABLE 2
MONTE CARLO SIMULATION VS EXACT VALUES OF CDF

x	n=3		n=4		n=5	
	MC	EXACT	MC	EXACT	MC	EXACT
.40	.7883	.7867	.7624	.7625	.7607	.7600
.45	.8604	.8592	.8725	.8725	.8671	.8693
.50	.9182	.9167	.9396	.9375	.9405	.9405
.55	.9598	.9592	.9694	.9680	.9768	.9778
.60	.9864	.9867	.9860	.9865	.9915	.9920
.65	.9989	.9991	.9952	.9960	.9975	.9975

TABLE 3

CRITICAL VALUES FOR F_n TEST
OBTAINED BY MONTE CARLO SIMULATION

Sample Size	20	30	50
Significance Level			
.10	.42714	.41903	.40816
.05	.44865	.43553	.42116
.01	.48659	.46579	.44487

TABLE 4

EMPIRICAL SIGNIFICANCE LEVEL OF
FOUTZ F_n TEST USING MONTE CARLO APPROXIMATION
(225,000 REPLICATIONS)

Sample Size	20	30	50
Significance Level			
.10	.1006	.0970	.1003
.05	.0486	.0486	.0498
.01	.0103	.0101	.0102

3. DESCRIPTION OF SIMULATION

In our simulation we generated deviates from three families of distributions. The family of asymmetric stable distributions has been previously used by Saniga and Miles (1979) to investigate the power of several goodness-of-fit tests. We used the same set of parameter values, $\alpha = 1.0(.3)1.9$ and $\beta = 0(.25)1.00$ they used. Mixed normal distributions have often been used for such tests, and we have included a family which is a composite of $N(0,1)$ and $N(0,\sigma)$, $\sigma \neq 1$, and another set which is a composite of $N(.5,1)$ and $N(0,\sigma)$. Pearson distributions considered include a variety of shapes, from "pear" normal to U and J shaped. Discussion of the procedures used to generate pseudorandom deviates from each of these families follows.

The sample data was obtained by starting with one or more uniformly distributed pseudorandom deviates. These were generated using the IMSL subroutine GGUBS, the basis of which is discussed in Lewis, Goodman, and Miller (1969).

3.1 Generation of Asymmetric Stable Deviates (Random Stabilized Standard Form)

This family of distributions contains as a special case the normal distribution ($\alpha = 2$, $\beta = 0$) and the Cauchy distribution ($\alpha = 1$, $\beta = 0$). For $\alpha < 2$ the distributions have infinite variance, which makes them useful for determining the ability of a goodness-of-fit test to detect heavy tailed distributions. The deviates were generated using the program given in Chambers, Mallows, and Stuck (1976). This subroutine, RSTAB, used one deviate from $U[0,1]$ and one exponentially distributed deviate to generate one RSSF deviate.

3.2 Generation of Mixed Normal Deviates

Mixed normals of the form $(1-\gamma)N(\mu_1, \sigma_1^2) + \gamma N(\mu_2, \sigma_2^2)$ were generated using the IMSL routine MDNRIS to convert uniform samples into standard normal samples. To obtain a set of N mixed normal variates we proceeded as follows:

(i) $2N$ uniform random variates $\{u_i\}$ were generated using GGUBS; (ii) for each $i = 1, \dots, N$, MDNRIS was used to convert u_i to a standard normal z_i . z_i was then transformed to a normal with mean μ_1 and variance σ_1^2 , or with mean μ_2 and variance σ_2^2 , depending on whether $u_{i+N} > \gamma$, or $u_{i+N} < \gamma$, respectively.

3.3 Generation of Pearson Type I, II Deviates

The generation of samples from Pearson Type I and II distributions was done via table look up and linear interpolation on the inverse cumulative distribution function. Sufficient entries to assure four significant decimal places in the final answer was achieved adaptively using numerical integration. Before discussion of the precise details of the process, we digress for a discussion of the Pearson distributions, and particularly types I and II.

Following Johnson and Kotz (1970), the Pearson probability density function $p(x)$ is given by

$$\frac{1}{p} \frac{dp}{dx} = \frac{a+x}{c_0 + c_1 x + c_2 x^2} .$$

It can be shown that a , c_0 , x_1 , and c_2 can be expressed in terms of non-negative parameters β_1 , β_2 , and the variance μ_2 , obtaining

$$\begin{aligned} c_0 &= (4\beta_2 - 3\beta_1)(10\beta_2 - 12\beta_1 - 18)^{-1}\mu_2 \\ a &= c_1 = \sqrt{\beta_1} (\beta_2 + 3)(10\beta_2 - 12\beta_1 - 18)^{-1} \sqrt{\mu_2} \\ c_2 &= (2\beta_2 - 3\beta_1 - 6)(10\beta_2 - 12\beta_1 - 18)^{-1}. \end{aligned} \quad (7)$$

Type I distributions are characterized by

$$\kappa := \frac{1}{4} c_1^2 (c_0 c_2)^{-1} < 0 ,$$

while Type II have $\kappa = 0$ with $\beta_1 = 0$, $\beta_2 < 3$. For Type I and II the probability density function is of the form

$$p(x) = K(x - a_1)^{m_1} (a_2 - x)^{m_2} . \quad (8)$$

Generation of a sequence of pseudorandom Pearson Type I and II deviates involves the following steps: (i) generate a sequence of deviates $\{u_i\}$, from $U[0,1]$; (ii) transform these to Pearson deviates by finding v_i so that $\int_{a_1}^{v_i} p(x)dx = u_i$. This necessitates being able to efficiently obtain the inverse CDF, i.e., if $F(v) = \int_{a_1}^v p(x)dx$, then we need $v_i = F^{-1}(u_i)$. We will denote F^{-1} by G in order to simplify notation.

Representation of the inverse CDF, $G(s)$, was achieved by linear interpolation in a table generated by numerical integration of $p(x)$. The adaptive process used to assure four decimal place accuracy, i.e., magnitude of the error less than $.5 \times 10^{-4}$ is described now.

The error in linear interpolation between points (s_i, G_i) and (s_{i+1}, G_{i+1}) is no more than $\frac{1}{8}(s_{i+1} - s_i)^2 G_2$, where $G_2 = \max_{s_i < s < s_{i+1}} |G''(s)|$.

Since the intervals will be small, we can approximate

$$G_2 \text{ by } \frac{G'(s_{i+1}) - G'(s_i)}{s_{i+1} - s_i} , \quad \text{and then using the fact that}$$

$$G'(s) = (F^{-1}(s))' = \frac{1}{F'(G)} = \frac{1}{p(G)} , \text{ we obtain}$$

$$G_2 \approx \frac{\frac{1}{p(G_{i+1})} - \frac{1}{p(G_i)}}{s_{i+1} - s_i} .$$

Thus, an error estimate in (s_i, s_{i+1}) is given by

$$\left| \frac{\frac{1}{p(G_{i+1})} - \frac{1}{p(G_i)}}{s_{i+1} - s_i} \right| \left| \frac{(s_{i+1} - s_i)^2}{8} \right| = \frac{1}{8} \left| \frac{1}{p(G_{i+1})} - \frac{1}{p(G_i)} \right| (s_{i+1} - s_i) . \quad (9)$$

Potential problems occur if $p(G) = 0$, as may happen at a_1 and a_2 . Since we require error less than $.5 \times 10^{-4}$, it is clear this must be the case when $|G_{i+1} - G_i| < .5 \times 10^{-4}$; hence if $p(G)$ is very small in the interval (s_i, s_{i+1}) we have an alternative scheme for accepting an interval, one which came into play for ranges where p is small.

These ideas were the basis for adaptive construction of a suitable table (s_i, G_i) for the inverse cumulative distribution function. We have

$s_0 = 0$, $G_0 = a_1$. We describe the general scheme for obtaining (s_{i+1}, G_{i+1}) given (s_i, G_i) and note how an estimate of G_{i+1} is generated afterwards. Given an estimate for G_{i+1} , the value of $\Delta s_i = s_{i+1} - s_i$ is obtained by numerical numerical integration of $\int_{G_i}^{G_{i+1}} p(x)dx$. This is accomplished by the adaptive quadrature routine, DCADRE, from the IMSL library. If m_1 or m_2 are negative, subtracting out the singularity was used for intervals near a_1 and a_2 , respectively. The double precision version of DCADRE is used and an absolute error tolerance of 10^{-6} is requested. The routine returns Δs_i and the error estimate (9) is computed. If it is less than $.5 \times 10^{-4}$, the result is accepted, we set $s_{i+1} = s_i + \Delta s_i$, and proceed to the next interval. Suppose the error estimate, $E_{\text{est}} > .5 \times 10^{-4}$. Since $E_{\text{est}} = O((\Delta G)^2)$, and

$O(\Delta G_i) = O(\Delta s_i)$, we obtain a new estimate for ΔG_i by taking it to be

$$\frac{\Delta G_i}{2} \left(\sqrt{\frac{.25 \times 10^{-4}}{E_{\text{est}}}} + 1 \right).$$

This takes the error based on the new ΔG_i to approximately midway between its current value and $.25 \times 10^{-4}$. More than one correction of this sort may be required, in particular initially where a reasonable estimate of ΔG_0 is not available.

Once an interval has been accepted (or rather, a point (s_i, G_i)) we increment the interval counter i , and then estimate the new interval size

ΔG_i by $\frac{\Delta G_{i-1}}{2} \left(\sqrt{\frac{.475 \times 10^{-4}}{E_{\text{est}}}} + 1 \right)$. This yields, based on the above assumptions,

a ΔG_i which should give an error for the next interval which is midway between the previous one and $.475 \times 10^{-4}$.

An initial value for ΔG_0 is required. Although not very sophisticated, we simply take $\Delta G_0 = \frac{a_2 - a_1}{1000}$, and depend on the adaptive machinery described above to decrease it to meet the error tolerance, or increase successive intervals as required for efficient representation.

It is true that the final value of s_i , call it s_N , should be equal to one. Because of the numerical integration, this is never exactly the case. In the worst case, $\beta_1 = .01$, $\beta_2 = 1.9$, we obtained the final value $s_N \approx .999999845$ an error of about 1.55×10^{-8} . In order to avoid any problems due to the table not covering $[0,1]$ exactly the simplest procedure was to replace each computed s_i by s_i/s_N , thus distributing the error over the entire interval and yielding a consistent table. Note that this is well within the error tolerance of $.5 \times 10^{-4}$.

The procedure has been thoroughly tested for its efficiency in representing the inverse CDF as well as for accuracy. For the particular cases

in which we were interested, the inverse CDF was represented by a table with no more than 729 entries. This occurred for the case $\beta_1 = .25$, $\beta_2 = 3.2$, which has an inverse CDF with very large slopes. For more gently sloping inverse CDF's we were able to use as few as 101 intervals, as in the case $\beta_1 = .01$, $\beta_2 = 1.75$. Most intervals had an error estimate of between $.4 \times 10^{-4}$ and $.5 \times 10^{-4}$, which shows that the interval sizing process we have used worked quite efficiently, with few initial estimates being rejected for being too large, without also resulting in intervals much too small.

The routine was checked against the published tables of Johnson, Nixon, Amos and Pearson (1963) for many values of the parameters and at most of the percentage points. With a few exceptions, where a difference of one in the fourth decimal place was noted, the results check exactly. Generally in these cases the fifth place was four or five so that the actual error was probably well within our tolerance.

4. RESULTS

The results of the simulation are summarized in Tables 5-12. The empirical power, in 5000 replications, of the Foutz F_n test compared with that for the Chi squared test or the K-S test is presented as a percent improvement in the probability of rejecting the null hypothesis that the distribution of the samples is the standard normal. A negative entry means that the power of the F_n test was smaller than that for the Chi squared test or the K-S test, whichever is appropriate.

The simulation has revealed that the F_n test is better than the Chi squared test which in turn is better than the K-S test when the true distribution of the samples is heavy tailed. Many such distributions are included in the asymmetric stable family as well as the family of mixtures of normals. For the mixed normal family, if the two normals involved in the mixture differ in the means the K-S test performed better than the F_n test even when the variances differed. We now discuss in more detail the results for each of the three families of distributions.

4.1 Asymmetric Stable Family

The results for the F_n test versus the Chi squared test are summarized in Table 5. The F_n test outperformed the Chi squared test for n equal to 21 and 31. When $n = 51$, as $\alpha + \beta$ increased the performance of the F_n test deteriorated as can be seen from the lower right part of Table 5. Another general observation is that as the significance level is decreased the improvement in power for the F_n test is accentuated.

The comparative figures for the F_n test versus the K-S test are presented in Table 6. Here again the F_n test did much better than the K-S test;

the results also indicate that for the asymmetric stable family the Chi squared test has a higher power than the K-S test.

4.2 Mixtures of Normals

The mixed normal distributions that we considered were of two basic types. The first type is of the form $(1-\gamma)N(0,1) + \gamma N(0,\sigma)$ with $\sigma = 2, 3, 4$ and $\gamma = .1, .2, .3, 1.0$; note that when $\gamma = 1.0$ the distribution is not a true mixture but $N(0,\sigma)$. The second type is a mixture of the form $(1-\gamma)N(.5,1) + \gamma N(0,\sigma)$ with $\sigma = 3, \gamma = .2, .3$ and $\sigma = 4, \gamma = .2$.

The F_n test did significantly better than the Chi squared test except for $n = 51$ and $\gamma = 1.0$ (see Table 7); in the latter case the Chi squared test turned out to be the better of the two tests.

When the F_n test was compared with the K-S test the F_n test turned out to be consistently better (Table 8).

In the case of a mixture of the second type, which included a location shift (Tables 9, 10), the K-S test proved to be superior to the F_n test while the comparison between the F_n test and the Chi squared test appeared to be inconclusive.

4.3 Pearson Family

We chose ten distributions of types I and II to encompass a variety of shapes as shown in figure 1; the standard normal is superimposed as a dotted curve in each of the graphs. The comparison of the F_n test versus the Chi squared test proved to be inconclusive. However, the K-S test appeared to have a higher power than the F_n test when the shape of the distribution was "near" normal such as the ones for $(\beta_1, \beta_2) = (0, 2.3), (0, 2.8)$ and $(.25, 3.2)$.

FIGURE 1

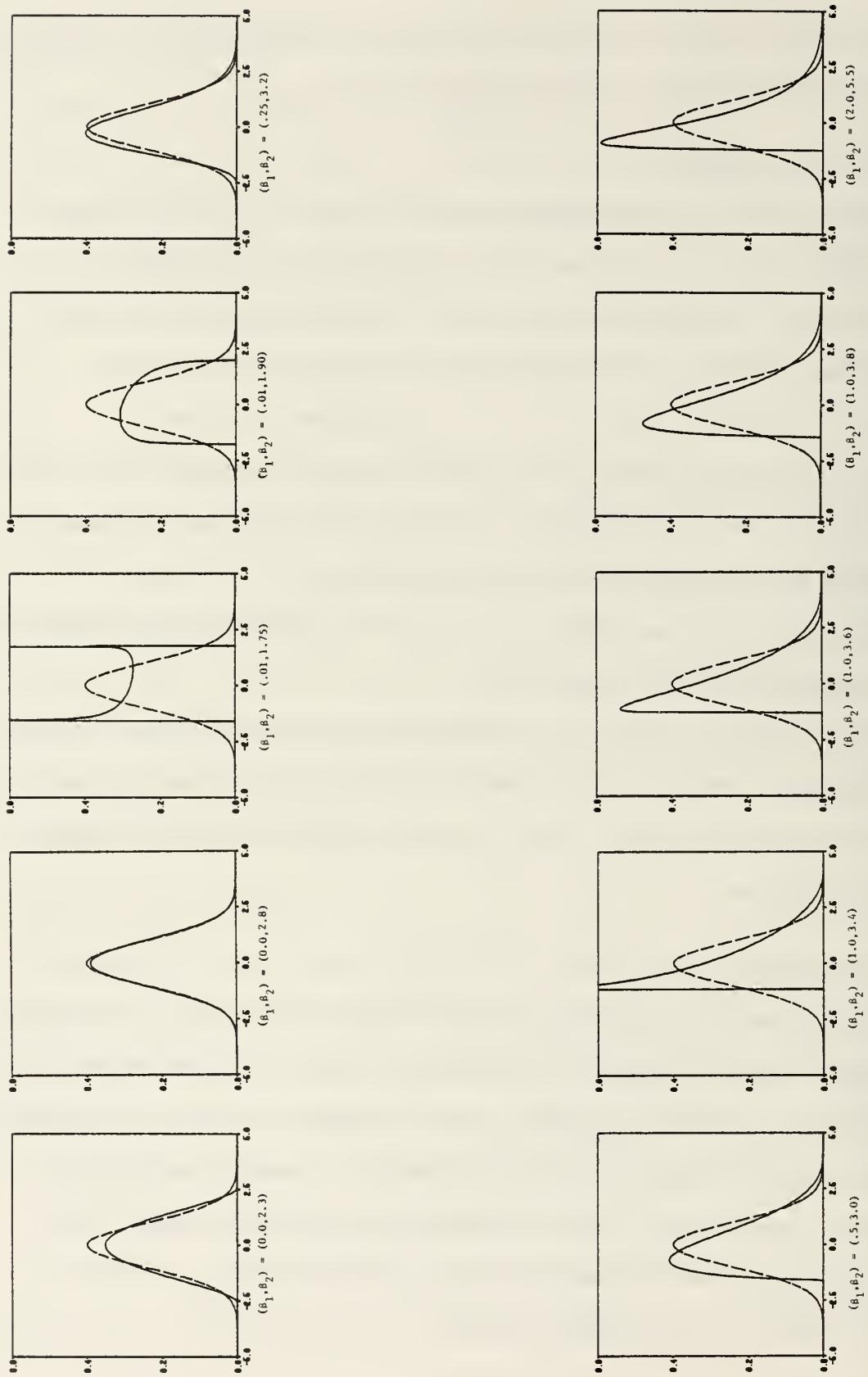


TABLE 5

FOUTZ F_n TEST VS CHI SQUARED TEST

RANDOM STABILIZED STANDARD DISTRIBUTIONS

	Sig Level	$\beta \rightarrow$	0.0	0.25	0.50	0.75	1.00
			α				
n=21	.10	1.0	102.0	102.7	101.6	95.8	96.9
		1.3	96.1	88.6	69.1	49.1	11.7
		1.6	72.0	68.3	59.5	49.0	29.2
		1.9	54.3	55.4	55.4	53.9	55.8
	.05	1.0	120.2	117.2	121.6	114.9	110.3
		1.3	97.9	98.2	71.2	49.6	- 1.4
		1.6	71.2	63.4	58.6	41.4	20.1
		1.9	49.8	45.6	51.1	49.8	44.2
	.01	1.0	256.0	215.9	238.0	234.5	233.8
		1.3	174.8	158.6	131.8	71.7	- 8.0
		1.6	118.5	130.3	107.9	62.1	21.7
		1.9	81.6	73.0	99.0	89.7	78.7
n=31	.10	1.0	37.9	36.8	38.1	39.3	37.0
		1.3	35.9	34.0	28.4	13.8	-10.4
		1.6	26.6	26.2	16.5	11.1	0.9
		1.9	17.6	19.5	19.4	13.4	16.7
	.05	1.0	47.4	44.6	46.6	50.2	46.7
		1.3	41.8	38.9	31.5	11.7	-18.6
		1.6	28.8	25.7	13.0	5.2	- 7.6
		1.9	12.0	15.8	13.3	8.4	14.4
	.01	1.0	74.7	75.2	73.8	73.5	70.2
		1.3	55.8	46.2	31.2	3.9	-33.7
		1.6	32.2	30.7	11.0	- 6.0	-21.3
		1.9	14.5	18.1	10.1	8.0	1.0
n=51	.10	1.0	11.4	13.3	11.1	10.6	11.4
		1.3	9.6	12.3	6.1	- 1.1	-11.1
		1.6	1.2	- 0.5	- 4.9	- 7.3	-13.9
		1.9	-10.8	- 8.9	-11.0	-11.1	-11.2
	.05	1.0	17.4	18.5	17.1	16.3	17.1
		1.3	12.6	12.2	5.0	-3.5	-18.2
		1.6	0.5	- 3.5	- 8.7	-12.0	-21.7
		1.9	-14.3	-12.9	-15.1	-16.3	-15.9
	.01	1.0	30.5	30.5	31.2	28.4	25.8
		1.3	14.1	13.8	3.1	-12.2	-35.2
		1.6	- 5.4	- 8.3	-20.6	-23.5	-37.0
		1.9	-24.2	-24.2	-24.1	-24.2	-26.1

TABLE 6

FOUTZ F TEST VS KOLMOGOROV-SMIRNOV TEST RANDOM
RANDOM STABILIZED STANDARD DISTRIBUTIONS

		$\beta \rightarrow$	0.0	0.25	0.50	0.75	1.00
		Sig Level	α				
n=21	.10	1.0	120.2	123.8	110.0	120.4	115.2
		1.3	102.8	100.6	76.0	43.5	0.9
		1.6	87.7	77.2	61.3	49.6	22.7
		1.9	65.5	61.7	62.9	65.1	57.8
	.05	1.0	197.5	196.0	186.7	185.9	192.1
		1.3	170.8	151.2	111.3	68.1	- 2.8
		1.6	131.3	110.4	89.9	67.0	30.4
		1.9	83.7	90.5	87.5	85.6	74.4
	.01	1.0	601.5	465.5	530.4	522.7	526.8
		1.3	396.4	338.1	245.0	121.4	- 0.6
		1.6	305.7	243.9	165.2	117.7	45.6
		1.9	140.1	147.5	176.2	176.5	163.8
n=31	.10	1.0	82.1	80.6	83.3	84.7	81.6
		1.3	83.6	77.6	56.7	25.5	- 9.2
		1.6	63.8	59.5	42.6	29.4	7.3
		1.9	49.5	55.6	48.2	44.8	40.3
	.05	1.0	150.9	149.8	145.6	152.7	151.5
		1.3	147.3	129.2	88.1	40.5	- 13.0
		1.6	107.8	98.9	65.0	41.1	11.4
		1.9	77.2	86.7	69.5	72.5	68.2
	.01	1.0	428.5	429.5	447.0	399.0	471.3
		1.3	386.1	316.4	190.6	76.4	- 16.2
		1.6	267.3	210.3	136.9	79.9	24.1
		1.9	190.8	186.2	153.0	146.8	136.8
n=51	.10	1.0	38.5	42.7	38.9	37.5	37.7
		1.3	49.3	49.4	29.6	8.4	- 10.3
		1.6	40.0	35.7	24.0	11.4	- 6.7
		1.9	24.4	31.4	26.1	22.2	19.5
	.05	1.0	77.6	81.2	78.0	78.3	76.0
		1.3	94.4	85.2	53.2	14.7	- 16.4
		1.6	79.4	61.4	37.9	17.0	- 8.1
		1.9	49.8	54.6	50.6	42.0	35.4
	.01	1.0	285.4	287.0	275.2	270.9	262.6
		1.3	287.6	243.0	138.7	37.7	- 26.1
		1.6	196.2	173.3	91.2	41.0	- 7.9
		1.9	130.4	126.3	115.4	111.8	99.6

TABLE 7

FOUTZ F_n TEST VS CHI SQUARED TEST
 MIXED NORMAL DISTRIBUTIONS

		$\gamma \rightarrow$	1.00	0.30	0.20	0.10
	Sig Level	N(0,σ)				
n=21	.10	N(0,2)	55.8	69.3	76.3	40.7
		N(0,3)	32.8	86.8	75.9	44.7
		N(0,4)	20.5	101.4	89.0	75.7
	.05	N(0,2)	47.0	46.4	64.5	23.7
		N(0,3)	30.5	71.3	69.0	34.7
		N(0,4)	22.6	91.1	76.8	48.1
	.01	N(0,2)	62.0	262.1	107.5	88.6
		N(0,3)	61.2	109.8	145.8	86.3
		N(0,4)	49.0	118.3	131.1	125.0
n=31	.10	N(0,2)	13.8	46.9	42.4	22.0
		N(0,3)	4.3	56.7	50.6	51.5
		N(0,4)	1.8	55.8	58.7	39.2
	.05	N(0,2)	12.0	38.3	29.2	28.6
		N(0,3)	3.1	53.2	48.4	53.2
		N(0,4)	1.0	57.4	61.5	44.4
	.01	N(0,2)	1.2	62.1	46.0	52.1
		N(0,3)	0.6	47.3	103.1	104.7
		N(0,4)	0.1	84.0	95.5	61.5
n=51	.10	N(0,2)	-13.1	23.6	26.1	9.7
		N(0,3)	-5.9	19.3	28.4	25.7
		N(0,4)	-1.4	26.2	29.5	35.6
	.05	N(0,2)	-16.2	28.9	45.5	17.4
		N(0,3)	-9.4	29.2	43.4	38.0
		N(0,4)	-2.5	34.1	45.4	39.7
	.01	N(0,2)	-26.9	83.8	54.8	67.4
		N(0,3)	-17.7	41.6	45.9	67.9
		N(0,4)	-7.9	24.5	78.3	51.4

TABLE 8

FOUTZ F_n TEST VS KOLMOGOROV-SMIRNOV TEST
 MIXED NORMAL DISTRIBUTIONS

		$\gamma \rightarrow$	1.00	0.30	0.20	0.10
	Sig Level	$N(0, \sigma)$				
$n=21$.10	$N(0, 2)$	60.5	36.1	31.8	8.7
		$N(0, 3)$	52.6	59.3	42.9	19.3
		$N(0, 4)$	38.8	71.5	54.0	40.2
	.05	$N(0, 2)$	87.4	49.5	36.4	6.8
		$N(0, 3)$	91.8	75.4	49.8	16.9
		$N(0, 4)$	74.4	90.2	66.6	43.7
	.01	$N(0, 2)$	156.3	94.4	43.1	15.8
		$N(0, 3)$	223.6	112.3	71.0	41.8
		$N(0, 4)$	198.1	107.0	101.2	26.6
$n=31$.10	$N(0, 2)$	42.6	19.3	27.8	13.9
		$N(0, 3)$	31.3	52.9	38.8	24.1
		$N(0, 4)$	18.5	66.2	52.9	21.1
	.05	$N(0, 2)$	69.3	29.9	27.8	20.6
		$N(0, 3)$	60.8	65.7	52.9	36.8
		$N(0, 4)$	38.7	101.4	61.9	36.2
	.01	$N(0, 2)$	135.8	52.9	67.3	55.3
		$N(0, 3)$	183.9	107.5	120.3	66.0
		$N(0, 4)$	129.8	163.9	152.2	72.1
$n=51$.10	$N(0, 2)$	22.4	26.8	23.6	9.7
		$N(0, 3)$	9.6	47.8	39.4	21.2
		$N(0, 4)$	3.1	66.4	51.1	36.3
	.05	$N(0, 2)$	45.7	30.0	39.9	9.0
		$N(0, 3)$	24.5	74.0	52.2	29.9
		$N(0, 4)$	10.6	110.7	73.3	52.3
	.01	$N(0, 2)$	117.6	61.9	81.1	20.3
		$N(0, 3)$	96.0	215.7	83.3	39.1
		$N(0, 4)$	53.0	170.9	164.5	86.7

TABLE 9

FOUTZ F_n TEST VS CHI SQUARED TEST
 MIXED NORMAL DISTRIBUTIONS

	MIXED NORMAL	.10	.05	.01
n=21	0.70xN(0.5,1)+0.30xN(0.0,3)	24.0	- 0.7	-20.8
	0.80xN(0.5,1)+0.20xN(0.0,3)	14.4	- 9.2	-27.2
	0.80xN(0.5,1)+0.20xN(0.0,4)	17.2	- 3.2	-21.1
n=31	0.70xN(0.5,1)+0.30xN(0.0,3)	9.6	- 6.8	-28.6
	0.80xN(0.5,1)+0.20xN(0.0,3)	3.3	-12.4	-33.3
	0.80xN(0.5,1)+0.20xN(0.0,4)	3.7	-10.7	-30.2
n=51	0.70xN(0.5,1)+0.30xN(0.0,3)	25.1	-24.1	-43.6
	0.80xN(0.5,1)+0.20xN(0.0,3)	- 2.3	-23.6	-48.6
	0.80xN(0.5,1)+0.20xN(0.0,4)	- 4.1	-26.3	-45.5

TABLE 10

FOUTZ F_n TEST VS KOLMOGOROV-SMIRNOV TEST
 MIXED NORMAL DISTRIBUTIONS

		SIGNIFICANCE LEVEL	.10	.05	.01
	MIXED NORMAL				
n=21	0.70xN(0.5,1)+0.30xN(0.0,3)		-17.0	-28.9	-33.7
	0.80xN(0.5,1)+0.20xN(0.0,3)		-27.3	-38.2	-40.7
	0.80xN(0.5,1)+0.20xN(0.0,4)		-25.5	-32.5	-35.1
n=31	0.70xN(0.5,1)+0.30xN(0.0,3)		-25.4	-34.5	-43.4
	0.80xN(0.5,1)+0.20xN(0.0,3)		-34.8	-45.5	-50.7
	0.80xN(0.5,1)+0.20xN(0.0,4)		-34.6	-40.2	-45.9
n=51	0.70xN(0.5,1)+0.30xN(0.0,3)		-28.7	-47.6	-56.7
	0.80xN(0.5,1)+0.20xN(0.0,3)		-48.7	-57.4	-64.9
	0.80xN(0.5,1)+0.20xN(0.0,4)		-43.3	-51.3	-61.0

TABLE 11

FOUTZ F_n TEST VS CHI SQUARED TEST
 PEARSON DISTRIBUTIONS

			SIGNIFICANCE LEVEL		
			.10	.05	.01
n=21	0.0	2.30	3.4	-10.5	-20.6
	0.0	2.80	21.1	-0.9	-12.4
	0.01	1.75	12.8	41.1	57.8
	0.01	1.90	-9.1	-2.4	18.9
	0.25	3.20	16.6	-0.4	-2.3
	0.50	3.00	17.5	5.8	-9.3
	1.00	3.40	-12.5	-16.2	-15.8
	1.00	3.60	9.1	-31.4	-30.9
	1.00	3.80	16.1	-17.6	-24.8
	2.00	5.50	-16.1	-14.9	-20.7
n=31	0.0	2.30	-6.6	-19.0	-22.5
	0.0	2.80	10.7	-12.1	-5.4
	0.01	1.75	-5.3	39.4	83.5
	0.01	1.90	-18.7	-6.9	26.9
	0.25	3.20	0.7	-11.8	-3.6
	0.50	3.00	-1.1	-0.6	-12.2
	1.00	3.40	-25.5	-23.5	-23.4
	1.00	3.60	-5.7	-41.4	-41.7
	1.00	3.80	0.0	-26.9	-32.9
	2.00	5.50	-29.5	-19.4	-26.5
n=51	0.0	2.30	30.4	-39.0	-19.7
	0.0	2.80	34.9	-40.0	-17.3
	0.01	1.75	6.6	68.1	132.9
	0.01	1.90	-11.8	-15.8	20.7
	0.25	3.20	33.3	-10.8	-15.6
	0.50	3.00	2.5	-19.9	-20.2
	1.00	3.40	-37.4	-35.7	-35.6
	1.00	3.60	-13.6	-53.7	-56.7
	1.00	3.80	-4.7	-40.6	-46.5
	2.00	5.50	-38.5	-28.7	-38.0

TABLE 12

FOUTZ F_n TEST VS KOLMOGOROV-SMIRNOV TEST
 PEARSON DISTRIBUTIONS

			SIGNIFICANCE LEVEL		
			.10	.05	.01
n=21	0.0	2.30	-22.8	-25.0	-26.2
	0.0	2.80	5.4	-18.8	- 7.2
	0.01	1.75	- 9.4	20.6	66.0
	0.01	1.90	-33.3	-15.3	16.5
	0.25	3.20	- 8.1	-14.8	- 4.1
	0.50	3.00	9.9	21.7	34.9
	1.00	3.40	84.8	94.4	95.9
	1.00	3.60	52.9	64.5	76.7
	1.00	3.80	38.4	44.0	59.4
	2.00	5.50	62.4	74.7	70.4
n=31	0.0	2.30	-28.9	-36.8	-36.9
	0.0	2.80	11.2	-20.8	-13.0
	0.01	1.75	-14.1	27.6	89.1
	0.01	1.90	-36.7	-15.4	19.4
	0.25	3.20	-12.9	-14.0	- 9.3
	0.50	3.00	- 0.6	25.0	31.3
	1.00	3.40	100.0	134.3	131.7
	1.00	3.60	55.5	79.4	91.8
	1.00	3.80	39.9	46.2	65.3
	2.00	5.50	63.3	91.0	100.0
n=51	0.0	2.30	- 9.1	-48.6	-40.0
	0.0	2.80	- 4.9	-38.9	-35.8
	0.01	1.75	-18.4	62.0	171.8
	0.01	1.90	-37.4	-22.9	26.6
	0.25	3.20	- 8.1	- 9.6	-13.8
	0.50	3.00	0.0	25.9	47.0
	1.00	3.40	169.9	225.8	245.5
	1.00	3.60	64.9	138.9	154.4
	1.00	3.80	42.1	69.0	85.2
	2.00	5.50	103.0	151.0	164.7

5. CONCLUDING REMARKS

The superior performance of the Foutz F_n test in detecting certain types of deviations from the hypothesized distribution leads to several more problems to be considered. Of primary importance is the generation of percentage points for the distribution of F_n for various values of n . The intractability of the problem of obtaining the exact distribution requires an empirical approach to finding a correction to the asymptotic approximation given by Foutz.

Since the test is also applicable to p -variate distributions, an investigation of ways to obtain the statistically equivalent blocks, and then the probability content of them, at least for $p = 2$, is to be considered. The problems of obtaining these blocks and their contents becomes increasingly complicated in higher dimensions.

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APPENDIX 1

THIS PROGRAM GENERATES DEVIATES IN RANDOM STABILIZED STANDARD FORM BLOCKS OF 20, 30, AND 50. DEVIATES ARE CONSIDERED BY GENERATING 50, THEN CONSIDERING THE FIRST 20, THE NEXT 30, AND ALL 50. THE HYPOTHESIS THAT THEY CAME FROM A NORMAL DISTRIBUTION WITH MEAN AND VARIANCE AS GIVEN IN INPUT IS TESTED.

THE TESTS PERFORMED ARE THE CHI SQUARED, KOLMOGOROV-SMIRNOV, AND FOUTZ TEST AT THE CONFIDENCE LEVELS OF 10, 5, AND 1 PERCENT. THE INPUT VALUES ARE MEAN, VARIANCE, ISEED, AND THE NUMBER OF REPLICATIONS.

```

IMPLICIT REAL*8 (A-Z)
INTEGER*4 T,IER,N,I,NP1,N1,N2,N3,ISEED,NR,NSTOP,IDE,IQ,ISEED1
1 .NCEL(3),NST(3),NSMP(3),NCHI(3,3),NCHIN(3,3),NKS(3,3),
2 NKS(3,3),NFNN(3,3),NFN(3,3),NSR
DIMENSION RD(55),CHISQU(2),
1 KOLSMR(2),FOUTZL(2),FNTST(3,3),ZALF(3),RN(55),
REAL*4 CELLS(50),CCMP(50),Q,CHI,R(55),PDIF(6),QTST(3),RNN(55),
1 STST(3,3)
DATA STST/.26473,.21756,.16959,.29408,.24170,.18841,.35241,
1 .28987,.22604/
DATA NCEL/8,10,10/,NST/1,21,1/,NSMP/20,30,50/,QTST/.1,.05,.01/
1 ,CHISQU/'CHI SQUA:','RED TEST:','KOLSMR/'KOLMOG-S','MIR TEST:/
2 ,FOUTZL/'FOUTZ TE:','ST:','ZALC1,ZALC2/.243069D0,.367879D0/
DATA WT1,MEAN1,VARI1,WT2,MEAN2,VARI2/.8D0,0.D0,1.D0,.2D0,0.D0,3.D0/
3 ,ZALF/1.28155D0,1.64485D0,2.32E35D0/

```

THE NEXT 4 CARDS PUT IN THE EMPIRICAL CRITICAL VALUES FOR THE FOUTZ DISTRIBUTION.

```

DIMENSION FNEMP(3,3)
DATA FNEMP/.42714D0,.41903D0,.40816D0,
1 .44865D0,.43553D0,.42116D0,.48659D0,
2 .46579D0,.44487D0/

```

```

REAL*4 ALPHA,BETA,BPRIME
COMMON/NCRPRM/MEAN,VARI,SVARI
EXTERNAL NORM,UNIF
T = 1
PIB2 = CATAN(1.DC)*2.D0
100 PRINT 1
READ 2,MEAN,VARI
IF(MEAN+VARI.LE.0.D0)STOP
2 FORMAT(2E5.0)
SVARI = DSQRT(VARI)
WRITE(4,14)
14 FORMAT(1H1)
1 FORMAT('+INPUT MEAN AND VARIANCE, FORMAT(2E5.0)')
PRINT 17
READ 12,ALPHA,BETA
17 FORMAT('+ INPUT ALPHA AND BETA, FORMAT(2F5.0)')
12 FORMAT(5F5.0)
PRINT 3
READ 5,ISEED,NR
ISEED1 = ISEED
BPRIME = BETA
IF(ALPHA.EC.0.)GO TO 110
PHIZ = -PIB2*BETA*(1.-ABS(1.-ALPHA))/ALPHA
BPRIME = DTAN(PIB2*(1.-ALPHA))*DTAN(PHIZ*ALPHA)
110 CONTINUE
DSEED = DFLOAT(ISEED)
5 FORMAT(I10,I5)
3 FORMAT('+INPUT SEED AND NUMBER OF REPLICATIONS, FORMAT(I10,I5)')
DO 205 IQ=1,3
DO 205 N1=1,3
NCHI(N1,IQ) = 0
NCHIN(N1,IQ) = 0
NKS(N1,IQ) = 0
NKS(N1,IQ) = 0

```

```

NFN(N1,IC) = 0
FNTST(N1,IQ) = ZALC1*ZALF(IC)/DSQRT(DFLOAT(NSMP(N1)+1)) + ZALC2
THE FOLLOWING CARE PUTS IN THE EMPIRICAL VALUES FOR THE
FOUTZ TEST
FNTST(N1,IQ) = FNEMP(N1,IQ)

205 NFNN(N1,IQ) = 0
DO 300 I=1,NR
CALL RANSRV(50,RN,RD,CSEED,ALPHA,BPRIME)
DO 250 N1=1,3
N2 = NST(N1)
N3 = NSMP(N1)
CALL VSRTAD(RD(N2),N3)
CALL VSRTAD(RN(N2),N3)
DO 210 NSR=1,N3
RNN(NSR) = RN(NSR + N2-1)
210 R(NSR) = RD(NSR+N2-1)
IDF = 0
CALL GFIT(UNIF,NCEL(N1),RNN,N3,CELLS,COMP,CHI,IDF,Q,IER)
DO 241 IC=1,3
IF(Q.LT.QTST(IQ))NCHI(N1,IQ) = NCHI(N1,IQ) + 1
241 CONTINUE
CALL NKS1(UNIF,RNN,N3,PDIF,IER)
DO 242 IC = 1,3
IF(PDIF(1).GT.STST(N1,IQ))NKS(N1,IC) = NKS(N1,IQ) + 1
242 CONTINUE
CALL FGUTZ(UNIF,RNN,N3,Q)
DO 243 IC=1,3
IF(Q.GT.FNTST(N1,IQ))NFNN(N1,IQ) = NFNN(N1,IQ) + 1
243 CONTINUE
IDF = 0
CALL GFIT(NORM,NCEL(N1),R,N3,CELLS,COMP,CHI,IDF,Q,IER)
DO 244 IC = 1,3
IF(Q.LT.QTST(IQ))NCHIN(N1,IQ) = NCHIN(N1,IQ) + 1
244 CONTINUE
CALL NKS1(NORM,R,N3,PDIF,IER)
DO 245 IQ=1,3
IF(PDIF(1).GT.STST(N1,IQ))NKS(N1,IQ) = NKS(N1,IQ) + 1
245 CONTINUE
CALL FOUTZ(NORM,R,N3,Q)
DO 246 IC=1,3
IF(Q.GT.FNTST(N1,IC))NFNN(N1,IC) = NFNN(N1,IC) + 1
246 CONTINUE
250 CONTINUE
300 CONTINUE
ISEED = CSEED
WRITE(4,7)MEAN,VARI,NR,ISEED1,ISEED
WRITE(4,11)ALPHA,BETA
11 FORMAT(//' RANDOM STABLE STANDARDIZED FORM (RSSF), ALPHA =',F5.2,
1 ', BETA =',F5.2//)
1 WRITE(4,E)CHISQU,((NCHI(N1,IQ),
1 N1=1,3),IQ=1,3),((NCHIN(N1,IC),
2 N1=1,3),IQ=1,3)
1 WRITE(4,8)KOLSMR,((NKS(N1,IQ),N1=1,3),IQ=1,3),
1 ((NKS(N1,IC),N1=1,3),IQ=1,3)
1 WRITE(4,8)FOUTZL,((NFNN(N1,IQ),N1=1,3),IQ=1,3),((NFNN(N1,IQ),N1=1,
1 ),IQ=1,3)
7 FORMAT(//' THE VALUES OF MEAN (U) AND VARIANCE (V) ARE',2F12.5//)
9 ' THE RESULTS OF ',I5,' REPLICATIONS, STARTING WITH SEED',
1 I12//33X,'ENDING WITH NEXT SEC',I12//)
8 FORMAT(///20X,2A8//20X,'20 PTS',
2 6X,'30 PTS',6X,'50 PTS',//10X,'10%',3I12// CONTROL',2X,' 5%',
3 3I12/10X,' 1%',3I12// RSSF ',2X,'10%',3I12// VS',5X,' 5%',
4 3I12/' N (U,V)',2X,' 1%',3I12)
9 WRITE(4,9)((FNTST(N1,IQ),N1=1,3),IC=1,3)
9 FORMAT(/'0 THIS WAS BASED ON THE FOLLOWING TEST VALUES',
1 /(13X,3F12.5))
GO TO 100

```

```
END
```

```
SUBROUTINE RANSRV(ND,RAN,R,DSEED,ALPHA,BPRIME)
IMPLICIT REAL*8 (A-M,O-Z)
DIMENSION R(1),RAN(1)
REAL*4 RN(100),ALPHA,BPRIME,RSTAB,W
CALL GGUBS(DSEED,2*ND,RN)
DO 200 NN=1,ND
RAN(NN) = RN(NN)
W = -DLOG(1.00 - RN(NN+ND))
R(NN) = RSTAB(ALPHA,BPRIME,RN(NN),W)
200 CONTINUE
RETURN
END
```

```
SUBROUTINE NCRM(X,P)
REAL*8 MEAN,VARI,SVARI
COMMON/NCRPRM/MEAN,VARI,SVARI
T = (X - MEAN)/SVARI
P = .5*ERFC(-T*.7071068)
RETURN
END
```

```
SUBROUTINE UNIF(X,P)
P = X
RETURN
END
```

```
FUNCTION RSTAB(ALPHA,BPRIME,U,W)
C RSTAB IS A RANDOM STABLE STANDARDIZED FORM (WHATEVER)
C
C ARGUMENTS
C   ALPHA = CHARACTERISTIC EXPONENT
C   BPRIME = SKEWNESS IN REVISED PARAMETERIZATION
C   U      = UNIFORM VARIATE ON (0,1)
C   W      = EXPONENTIALLY DISTRIBUTED VARIATE
C DOUBLE PRECISION DA,DB
C DATA PIBY2/1.57079633/,PIBY4/.785398163/,THR1/.99/
C EPS = 1. - ALPHA
C COMPUTE SOME TANGENTS
C PIBY2 = PIBY2*(U -.5)
C A = PIBY2*TAN2(PIBY2)
C BB = TAN2(EPS*PHIEY2)
C B = EPS*PIBY2*BB
C IF(EPS.GT.-.99)TAU = BPRIME/(TAN2(EPS*PIBY2)*PIBY2)
C IF(EPS.LE.-.99)TAU = BPRIME*PIBY2*EPS*(1. - EPS)*TAN2((1. - EPS)*
1 PIBY2)
C COMPUTE SOME NECESSARY SUBEXPRESSIONS
C IF PHI NEAR PI BY 2, USE DOUBLE PRECISION.
C IF(A.GT.THR1)GO TO 50
C SINGLE PRECISION
C A2 = A**2
C A2P = 1. + A2
C A2 = 1. - A2
C B2 = B**2
C B2P = 1. + B2
C B2 = 1. - B2
C GO TO 100
C DOUBLE PRECISION
50  DA = DBLE(A)**2
DB = DBLE(B)**2
A2 = 1.00 - DA
A2P = 1.00 + DA
B2 = 1. - DB
B2P = 1. + DB
C COMPUTE COEFFICIENT
```

```

C 100 Z = A2P*(B2 + 2.*PHIBY2*BB*TAU)/(W*A2*B2P)
C COMPUTE THE EXPONENTIAL-TYPE EXPRESSION
C ALOGZ = ALOG(Z)
C D = D2(EPS*ALOGZ/(1. - EPS))*(ALOGZ/(1. - EPS))
C COMPUTE STABLE
C RSTAB = (1. + EPS*D)*2.*((A - B)*(1. + A*B) - PHIBY2*TAU*BB*(B*A2
C 1 - 2.*A))/(A2*B2P) + TAU*D
C RETURN
C END

C FUNCTION D2(Z)
C EVALUATE (EXP(X) - 1)/X
C DOUBLE PRECISION F1,F2,Q1,Q2,C3,PV,ZZ
C DATA P1,P2,Q1,Q2,Q3/.8400668525364E3239D3,.200011141589964569D2,
C 1.168013370507926648D4,.180013370407390023D3,1.D0/
C THE APPROXIMATION 1801 FOR HART ET AL (1968, P213)
C IF(ABS(Z).GT.0.1)GC TO 100
C ZZ = Z*Z
C PV = P1 + ZZ*P2
C D2 = 2.00*PV/(Q1 + ZZ*(Q2 + ZZ*Q3) - Z*PV)
C RETURN
C 100 D2 = (EXP(Z) - 1.)/Z
C RETURN
C END

C FUNCTION TAN(XARG)
C TANGENT FUNCTION
C LOGICAL NEG,INV
C DATA P0,P1,P2,Q0,Q1,Q2/.129221035E3,-.887662377E1,.528644456E-1,
C 1.164529332E3,-.45120561E2,1./
C THE APPRCXIMATION 4283 FROM HART ET AL (1968, P. 251)
C DATA PIBY4 /.785398163/,PIBY2/1.57079633/,PI/3.14159265/
C NEG = .FALSE.
C INV = .FALSE.
C X = XARG
C NEG = X.LT.0.
C X = ABS(X)
C PERFORM RANGE REDUCTION IF NECESSARY
C IF(X.LE.PIBY4)GC TC 50
C X = AMOD(X,PI)
C IF(X.LE.PIBY2)GO TO 30
C NEG = .NOT.NEG
C X = PI - X
C 30 IF(X.LE.PIBY4) GC TC 50
C INV = .TRUE.
C X = PIBY2 - X
C 50 X = X/PIBY4
C CONVERT TO RANGE OF RATIONAL
C XX = X*X
C TAN = X*(P0 + XX*(P1 + XX*P2))/(Q0 + XX*(Q1 + XX*Q2))
C IF(NEG)TAN = -TAN
C IF(INV)TAN = 1./TAN
C RETURN
C END

C FUNCTION TAN2(XARG)
C COMPUTE TAN(X)/X
C FUNCTION DEFINED ONLY FOR ABS(XARG).LE.PI BY 4
C FOR OTHER ARGUMENT RETURNS TAN(X)/X, COMPUTED DIRECTLY
C DATA P0,P1,P2,C0,C1,C2/.129221035E3,-.887662377E1,.528644456E-1,
C 1.164529332E3,-.45120561E2,1./
C THE APPRCXIMATION 4283 FROM HART ET AL (1968, P. 251)
C DATA PIBY4,PIBY2,PI/.785398163,1.57079633,3.14159265/
C X = ABS(XARG)
C IF(X.GT.PIBY4)GC TC 200
C X = X/PIBY4
C CONVERT TO RANGE OF RATIONAL

```

THIS PROGRAM GENERATES MIXED NORMAL DEVIATES $WT1*N(MEAN1, VAR1) + (1 - WT1)*N(MEAN2, VAR2)$.
 BLOCKS OF 20, 30, AND 50 DEVIATES ARE CONSIDERED BY GENERATING 50. THEN CONSIDERING THE FIRST 20, THE NEXT 30, AND ALL 50.
 THE HYPOTHESIS THAT THEY CAME FROM A NORMAL DISTRIBUTION WITH MEAN AND VARIANCE AS GIVEN IN INPUT IS TESTED.

THE TESTS PERFORMED ARE THE CHI SQUARED, KOLMOGOROV-SMIRNOV, AND FOUTZ TEST AT THE CONFIDENCE LEVELS OF 10, 5, AND 1 PERCENT.
 THE INPUT VALUES ARE MEAN, VARIANCE, ISEED, AND THE NUMBER OF REPLICATIONS.

```

IMPLICIT REAL*8 (A-Z)
INTEGER*4 T,IER,N,I,NP1,N1,N2,N3,ISEED,NR,NSTOP,IDEF,IQ,ISEED1
1 ,NCEL(3),NST(3),NSMP(3),NCHI(3,3),NCHIN(3,3),NKS(3,3),
2 NKSN(3,3),NFNN(3,3),NFN(3,3),NSR
DIMENSION RD(55),CHISQU(2),
1 KOLSMR(2),FOUTZL(2),FNTST(3,2),ZALF(3),RN(55),
REAL*4 CELLS(50),CCMP(50),Q,CFI,R(55),PDIF(6),QTST(3),RNN(55),
1 STST(3,3)
DATA STST/.26473,.21756,.16959,.29408,.24170,.18841,.35241,
1 .28987,.22604/
DATA NCEL/8,10,10/,NST/1,21,1/,NSMP/20,30,50/,QTST/.1,.05,.01/
1 ,CHISQU/'CHI SQUA','RED TEST',KOLSMR/'KOLMOG-S','MIR TEST'/
2 ,FOUTZL/'FOUTZ TE','ST',ZALC1,ZALC2/.243069D0,.367879D0/
DATA WT1,MEAN1,VAR1,WT2,MEAN2,VAR2/.8D0,0.0D0,1.D0,.2D0,0.D0,3.D0/
3 ,ZALF/1.28155D0,1.64485D0,2.32635D0/

```

THE NEXT 4 CARDS PUT IN THE EMPIRICAL CRITICAL VALUES FOR THE FCUTZ TEST.

```

DIMENSION FNEMP(3,3)
DATA FNEMP/.42714D0,.41903D0,.40816D0,
1 :44865D0,.43553D0,.42116D0,.48659D0,
2 :46579D0,.44487D0/

```

```

COMMON/NCRPRM/MEAN,VARI,SVARI
EXTERNAL NORM,UNIF
T = 1
100 PRINT 1
READ 2,MEAN,VARI
IF(MEAN+VARI.LE.0.)STOP
2 FORMAT(2E5.0)
SVARI = CSQRT(VARI)
WRITE(4,14)
14 FORMAT(1H1)
1 FORMAT('INPUT MEAN AND VARIANCE, FORMAT(2E5.0)')
PRINT 17
READ 12,WT1,MEAN1,VARI1,MEAN2,VARI2
WT2 = 1.D0 - WT1
17 FORMAT('INPUT WT1,MEAN1,VARI1,MEAN2,VARI2, FORMAT(5F5.0)')
12 FORMAT(5F5.0)
PRINT 3
READ 5,ISEED,NR
ISEED1 = ISEED
DSEED = DFLOAT(ISEED)
5 FORMAT(I10,I5)
3 FORMAT('INPUT SEED AND NUMBER OF REPLICATIONS, FORMAT(I10,I5)')
DO 205 IQ=1,3
DO 205 N1=1,3
NCHI(N1,IQ) = 0
NCHIN(N1,IQ) = 0
NKS(N1,IQ) = 0
NKSN(N1,IQ) = 0
NFN(N1,IQ) = 0
FNTST(N1,IQ) = ZALC1*ZALF(IQ)/DSQRT(DFLOAT(NSMP(N1)+1)) + ZALC2
FNTST(N1,IQ) = FNEMP(N1,IQ)
205 NFNN(N1,IQ) = 0
DO 300 I=1,NR

```

```

CALL MIXNRM(50,RN,RD,DSEED,WT1,MEAN1,VARI1,MEAN2,VAR2)
DO 250 N1=1,3
N2 = NST(N1)
N3 = NSMP(N1)
CALL VSRTAD(RD(N2),N3)
CALL VSRTAD(RN(N2),N3)
DO 210 NSR=1,N3
RNN(NSR) = RN(NSR + N2-1)
210 R(NSR) = RD(NSR+N2-1)
IDF = 0
CALL GFIT(UNIF,NCEL(N1),RNN,N3,CELLS,COMP,CHI,IDF,Q,IER)
DO 241 IQ=1,3
IF(Q.LT.QTST(IQ))NCHI(N1,IQ) = NCHI(N1,IQ) + 1
241 CONTINUE
CALL NKS1(UNIF,RNN,N3,PDIF,IER)
DO 242 IC = 1,3
IF(PDIF(1).GT.STST(N1,IQ))NKS(N1,IC) = NKS(N1,IQ) + 1
242 CONTINUE
CALL FOUTZ(UNIF,RNN,N3,Q)
DO 243 IQ=1,3
IF(Q.GT.FNTST(N1,IC))NFN(N1,IQ) = NFN(N1,IQ) + 1
243 CONTINUE
IDF = 0
CALL GFIT(NORM,NCEL(N1),R,N3,CELLS,COMP,CHI,IDF,Q,IER)
DO 244 IC = 1,3
IF(Q.LT.QTST(IQ))NCHIN(N1,IQ) = NCHIN(N1,IQ) + 1
244 CONTINUE
CALL NKS1(NORM,R,N3,PDIF,IER)
DO 245 IQ=1,3
IF(PDIF(1).GT.STST(N1,IQ))NKS(N1,IQ) = NKS(N1,IQ) + 1
245 CONTINUE
CALL FOUTZ(NORM,R,N3,Q)
DO 246 IQ=1,3
IF(Q.GT.FNTST(N1,IC))NFNN(N1,IQ) = NFNN(N1,IQ) + 1
246 CONTINUE
250 CONTINUE
300 CONTINUE
ISEED = DSEED
WRITE(4,7)MEAN,VARI,NR,ISEED1,ISEEDC
WRITE(4,11)WT1,MEAN1,VARI1,WT2,MEAN2,VAR2
11 FORMAT(//'* MIXED NORMAL (MIXNRM) = ',F5.2,'*N('',F3.1,'',',F3.1,
1 ') + ',F5.2, '*N('',F3.1,'',',F3.1.'')')
1 WRITE(4,8)CHISCU,((NCHI(N1,IQ),
1 N1=1,3),IQ=1,3),((NCHIN(N1,IC),
2 N1=1,3),IQ=1,3)
1 WRITE(4,8)KOLSMR,((NKS(N1,IQ),N1=1,3),IQ=1,3),
1 ((NKS(N1,IQ),N1=1,3),IQ=1,3)
1 WRITE(4,8)FOUTZL,((NFN(N1,IQ),N1=1,3),IQ=1,3),((NFNN(N1,IQ),N1=1
1 ),IQ=1,3)
7 FORMAT(//'* THE VALUES OF MEAN (U) AND VARIANCE (V) ARE',2F12.5//
9 ' THE RESULTS OF ',I5,' REPLICATIONS, STARTING WITH SEED',
1 I12//33X,'ENDING WITH NEXT SEED',I12//)
8 FORMAT(///20X,2A8//20X,'20 PTS',
2 6X,'30 PTS',6X,'50 PTS'//10X,'10%',3I12// CONTROL',2X,' 5%',
3 3I12/10X,' 1%',3I12// MIXNCPM',2X,'10%',3I12/' VS',5X,' 5%''
4 3I12/' N (U,V)',2X,' 1%',3I12)
9 WRITE(4,9)((FNTST(N1,IQ),N1=1,3),IC=1,3)
1 /'(13X,3F12.5)')
GO TO 100
END

```

```

SUBROUTINE MIXNRM(ND,RAN,R,DSEED,WT1,MEAN1,VARI1,MEAN2,VAR2)
IMPLICIT REAL*8 (A-N,C-Z)
DIMENSION R(1),RAN(1)
REAL*4 RN(100),RNN,RNNN
SVAR1 = DSQRT(VAF1)
SVAR2 = DSQRT(VAR2)
WT2 = 1.0 - WT1

```

```
XX = XX*  
TAN2 = (PO + XX*(P1 + XX*P2))/(PIBY4*(Q0 + XX*(Q1 + XX*Q2)))  
RETURN  
200 TAN2 = TAN(XARG)/XARG  
RETURN  
END
```

SUBROUTINE FOUTZ (PCCF,XT,NXT,FN)

THIS SUBROUTINE GENERATES THE STATISTIC FOR THE FOUTZ FN TEST.

INPUT VARIABLES ARE:

PCDF - THE CUMULATIVE DISTRIBUTION FUNCTION AGAINST WHICH THE DEVIATES ARE BEING TESTED. CALLING SEQUENCE MUST BE OF THE FORM 'CALL PCDF(X,P)', WHERE X IS AN INPUT VALUE, AND THE VALUE OF THE CUMULATIVE DISTRIBUTION FUNCTION IS RETURNED IN P.

P MUST BE BETWEEN 0 AND 1.

XT - THE ARRAY OF DEVIATES, IN INCREASING ORDER.

NXT - THE NUMBER OF DEVIATES IN THE ARRAY XT (= N - 1)

THE RETURNED VALUE IS FN, THE VALUE OF THE STATISTIC.

NXT IS PRESENTLY LIMITED TO A MAXIMUM OF 50 BY THE DIMENSION OF THE VARIABLE XTD.

DIMENSION XTD(1)

REAL*8 XTD(51),RN,FND

N = NXT + 1

DO 200 I=1,NXT

K = N - I

CALL PCDF(XT(K),P)

200 XTD(K+1) = P

RN = 1.00/A

XTD(1) = RN - XTD(2)

DO 300 I=2,NXT

300 XTD(I) = RN - XTD(I+1) + XTD(I)

XTD(N) = RN - 1.00 + XTD(N)

FND = 0.

DO 400 I=1,N

400 FND = FND + DMAX1(XTD(I),0.00)

FN = FND

RETURN

END

```

CALL GGUBS(DSEED,2*ND,RN)
DO 200 NN=1,ND
RAN(NN) = RN(NN)
CALL MDNRIS(RN(NN),RNN,IER)
IF(RN(NN+ND).GT.WT2)GO TO 150
R(NN) = RNN*SVAR2 + MEAN2
GO TO 200
150 R(NN) = RNN*SVAR1 + MEAN1
200 CONTINUE
RETURN
END

```

```

SUBROUTINE NCRM(X,P)
REAL*8 MEAN,VARI,SVARI
COMMON/NCRPRM/MEAN,VARI,SVARI
T = (X - MEAN)/SVARI
P = .5*ERFC(-T*.7071068)
RETURN
END

```

```

SUBROUTINE UNIF(X,P)
P = X
RETURN
END

```

SUBROUTINE FOUTZ (FCDF,XT,NXT,FN)
THIS SUBROUTINE GENERATES THE STATISTIC FOR THE FOUTZ FN TEST.
INPUT VARIABLES ARE:
PCDF - THE CUMULATIVE DISTRIBUTION FUNCTION AGAINST WHICH THE
DEVIATES ARE BEING TESTED. CALLING SEQUENCE MUST BE
THE FCRM 'CALL PCDF(X,P)', WHERE X IS AN INPUT VALUE
AND THE VALUE OF THE CUMULATIVE DISTRIBUTION FUNCTION
IS RETURNED IN P.
P MUST BE BETWEEN 0 AND 1.
XT - THE ARRAY OF DEVIATES, IN INCREASING ORDER.
NXT - THE NUMBER OF DEVIATES IN THE ARRAY XT (= N - 1)

THE RETURNED VALUE IS FN, THE VALUE OF THE STATISTIC.

```

NXT IS PRESENTLY LIMITED TO A MAXIMUM OF 50 BY THE DIMENSION OF
THE VARIABLE XTD.
DIMENSION XTD(1)
REAL*8 XTD(51),RN,FND
N = NXT + 1
DO 200 I=1,NXT
K = N - I
CALL PCDF(XT(K),P)
200 XTD(K+1) = P
RN = 1.00/N
XTD(1) = RN - XTD(2)
DO 300 I=2,NXT
300 XTD(I) = RN - XTD(I+1) + XTD(I)
XTD(N) = RN - 1.00 + XTD(N)
FND = 0.
DO 400 I=1,N
400 FND = FND + CMAX1(XTD(I),0.00)
FN = FND
RETURN
END

```

THIS PROGRAM GENERATES PEARSON TYPE I OR II RANDOM DEVIATES. THE PARAMETERS ARE CALCULATED IN TERMS OF B1 AND B2 IN SUBROUTINE PRM. SUBROUTINE ADINT1 IS USED TO CALCULATE THE CDF TABLE. GGUBS IS USED TO GENERATE RANDOM (0,1) DEVIATES (UNIFORMLY DISTRIBUTED), AND THEN SUBROUTINE RANDP1 DOES AND INVERSE CALCULATION TO OBTAIN THE RANDOM PEARSON DEVIATE.)

A SET OF 50 RANDOM DEVIATES ARE GENERATED, THEN THE CHI SQUARED TEST, THE KOLMOGOROV-SMIRNOV TEST, AND THE FOUTZ FN TEST ARE APPLIED. THE UNIFORM DEVIATES ARE TESTED AGAINST THE HYPOTHESIS THEY CAME FROM A UNIFORM DISTRIBUTION AS A CONTROL. THEN THE PEARSON DEVIATES ARE TESTED AGAINST THE HYPOTHESIS THAT THEY CAME FROM A NORMAL DISTRIBUTION. THIS TEST IS REPLICATED A NUMBER OF TIMES.

THE TESTS ARE APPLIED TO THE FIRST 20, THE NEXT 30, AND ALL 50.

THE SET OF DEVIATES IS ALSO TESTED AGAINST THE NORMAL DISTRIBUTION. THIS IS REPEATED FOR THE SAME SETS.

THE INPUT VALUES ARE B1,B2,ISEED, AND THE NUMBER OF REPLICATIONS

```

IMPLICIT REAL*8 (A-Z)
INTEGER*4 T,IER,N,I,NP1,N1,N2,N3,ISEED,NR,NSTOP,IDEF,IQ,ISEED1
1 ,NCEL(3),NST(3),NSMP(3),NCHI(3,3),NCHIN(3,3),NKS(3,3),
2 NKS(3,3),NFNN(3,3),NFFN(3,3),NAFFN(3,3),NAFN(3,3),NSR
DIMENSION X(2001),CDF(2001),A(2001),B(2001),RD(55),CHISQU(2),
1 KOLSMR(2),FOUTZL(2),ASFL(2),FNTST(3,3),ZALF(3),RN(55),
COMMON /PPARM/CO,C1,C2,A1,A2,BC0,BC1,M1,M2,KINV,XL,XR,MEAN,T
COMMON /(CDF/X,CDF,A,B,NP1
REAL*4 CELLS(50),CCMP(50),Q,CHI,R(55),PDIF(6),QTST(3),RNN(55),
1 STST(3,3)
DATA STST/.26473,.21756,.16959,.29408,.24179,.18841,.35241,
1 .28987,.22604/
DATA NCEL/8,10,10/,NST/1,21,1/,NSMP/20,30,50/,QTST/.1,.05,.01/
1 ,CHISQU/'CHI SQUA','RED TEST','KOLSMR/'KOLMOG-S','MIR TEST'/
2 ,FOUTZL/'EMP FOLT','Z TEST','ZALC1,ZALC2/.243069D0,.367879D0/
3 ,ZALF/1.28155D0,1.64485D0,2.32635D0/
      DIMENSION FNEMP(3,3)
      DATA FNEMP/.42714D0,.41903D0,.40816D0,
2           .44865D0,.43553D0,.42116D0,
3           .48659D0,.46579D0,.44487D0/
4 ,ASFL/'ASYM FOU','TZ TEST'/
EXTERNAL NCRM,UNIF
T = 1
100 PRINT 1
READ 2,B1,B2
IF(B1+B2.LE.0.0D0)STCP
2 FORMAT(2E5.0)
WRITE(4,14)
14 FORMAT(1H1)
1 FORMAT('+'B1 AND B2,FORMAT(2E5.0))
CALL PRM(B1,B2,1.0D0,IER)
IF(IER.NE.0)GO TO 900
A(1) = 0.0D0
CDF(1) = 0.
CALL INTSZ(N,X)
NP1 = N + 1
PRINT 3
READ 5,ISEED,NR
ISEED1 = ISEED
DSEED = DFLOAT(ISEED)
5 FORMAT(I10,I5)
3 FORMAT('+'INPUT SEED AND NUMBER OF REPLICATIONS,FORMAT(I10,I5))
CO 205 IQ=1,3
DO 205 N1=1,3
NCHI(N1,IQ) = 0
NCHIN(N1,IQ) = 0
NKS(N1,IC) = 0

```

```

NKS(N1,IQ) = 0
NFN(N1,IQ) = 0
FNTST(N1,IQ) = ZALC1*ZALF(IQ)/DSQRT(DFLOAT(NSMP(N1)+1)) + ZALC2
NAFNN(N1,IQ) = 0
NAFN(N1,IQ) = 0
205 NFNN(N1,IQ) = 0
DO 300 I=1,NR
CALL RANPDI(50,RN,RC,DSEED)
DO 250 N1=1,3
N2 = NST(N1)
N3 = NSMP(N1)
CALL VSRTAD(RD(N2),N3)
CALL VSRTAD(RN(N2),N3)
DO 210 NSR=1,N3
RNN(NSR) = RN(NSR + N2-1)
210 R(NSR) = RD(NSR+N2-1)
IDF = 0
CALL GFIT(UNIF,NCEL(N1),RNN,N3,CELLS,COMP,CHI,IDF,Q,IER)
DO 241 IQ=1,3
IF(Q.LT.QTST(IQ))NCHI(N1,IQ) = NCHI(N1,IQ) + 1
241 CONTINUE
CALL NKS1(UNIF,RNN,N3,PDIF,IER)
DO 242 IQ = 1,3
IF(PDIF(1).GT.STST(N1,IQ))NKS(N1,IQ) = NKS(N1,IQ) + 1
242 CONTINUE
CALL FOUTZ(UNIF,RNN,N3,Q)
DO 243 IC=1,3
IF(Q.GT.FNEMP(N1,IC))NFN(N1,IC) = NFN(N1,IC) + 1
IF(Q.GT.FNTST(N1,IC))NAFNN(N1,IC) = NAFNN(N1,IC) + 1
243 CONTINUE
IDF = 0
CALL GFIT(NORM,NCEL(N1),R,N3,CELLS,COMP,CHI,IDF,Q,IER)
DO 244 IQ = 1,3
IF(Q.LT.QTST(IQ))NCHIN(N1,IQ) = NCHIN(N1,IQ) + 1
244 CONTINUE
CALL NKS1(NORM,R,N3,PDIF,IER)
DO 245 IQ=1,3
IF(PDIF(1).GT.STST(N1,IQ))NKS(N1,IQ) = NKS(N1,IQ) + 1
245 CONTINUE
CALL FOUTZ(NORM,R,N3,Q)
DO 246 IQ=1,3
IF(Q.GT.FNEMP(N1,IC))NFNN(N1,IC) = NFN(N1,IC) + 1
IF(Q.GT.FNTST(N1,IC))NAFNN(N1,IC) = NAFNN(N1,IC) + 1
246 CONTINUE
250 CONTINUE
300 CONTINUE
ISEED = DSEED
WRITE(4,7)B1,B2,A1,A2,M1,M2,NR,ISEED1,ISEED
WRITE(4,8)CHISOU,((NCHI(N1,IC),
1 N1=1,3),IQ=1,3),((NCHIN(N1,IQ),
2 N1=1,3),IQ=1,3)
WRITE(4,8)KCLSMR,((NKS(N1,IQ),N1=1,3),IQ=1,3),
1 ((NKS(N1,IQ),N1=1,3),IQ=1,3)
WRITE(4,8)FOUTZL,((NFN(N1,IQ),N1=1,3),IQ=1,3),((NFNN(N1,IQ),N1=1,
1 ),IQ=1,3)
WRITE(4,8)ASFL,((NAFN(N1,IQ),N1=1,3),IQ=1,3),
1 ((NAFNN(N1,IC),N1=1,3),IQ=1,3)
7 FORMAT(// THE VALUES OF B1 AND B2 ARE',2F12.5///
A' VALUES OF A1, A2, M1, AND M2 ARE',//4F12.6///
9' THE RESULTS OF ',I5,' REPLICATIIONS, STARTING WITH SEED',
1 I12//3X,'ENDING WITH NEXT SEED',I12//)
8 FORMAT(//20X,2A8//20X,'20 PTS',
2 6X,'30 PTS',6X,'50 PTS',//10X,'10%',3I12// CONTROL',2X,' 5%',
3 3I12//10X,' 1%',3I12// PEARSON',2X,'10%',3I12// VS',5X,' 5%',
4 3I12// NORMAL',3X,' 1%',3I12)
GO TO 100
900 PRINT 4,N,IER,B1,B2,ERROR
4 FORMAT('N,IER,B1,B2',2I10,1P30.15.6)
STOP
END

```

```

SUBROUTINE RANPD1(NE,RAN,R,DSEED)
IMPLICIT REAL*8 (A-N,O-Z)
DIMENSION R(1),RAN(1)
REAL*4 RN(100),RNN
CALL GGUBS(DSEED,ND,RN)
DO 200 NN=1,ND
RAN(NN) = RN(NN)
CALL CDFINV(RAN(NN),RNN)
R(NN) = RNN
200 CONTINUE
RETURN
END

```

```

SUBROUTINE NORM(X,P)
P = .5*ERFC(-X*.7071068)
RETURN
END

```

```

SUBROUTINE UNIF(X,P)
P = X
RETURN
END

```

```

SUBROUTINE PRM(B1,B2,U2,IER)
IMPLICIT REAL*8 (A-Z)
COMMON /PPARM/C0,C1,C2,A1,A2,BIGCO,BIGC1,M1,M2,KINV,XL,XR,MEAN,T
INTEGER*4 T,IER
IER = 0
IF(T.EQ.3)B2 = (6. + 3.*B1)/2.
IF(T.EQ.5)B1 = 4.*((2.*B2 - 6.)*(4.*B2-3.)/((B2+3.)**2+
1_12.*((4.*B2-3.)))
DEN = 10.*B2 - 12.*B1 - 18.
C0 = (4.*B2 - 3.*B1)*U2/DEN
C1 = DSQRT(U2*B1)*(B2 + 3.)/DEN
C2 = (2.*B2 - 3.*B1 - 6.)/DEN
A1 = 0.
A2 = 0.
BIGCO = 0.
BIGC1 = 0.
M1 = 0.
M2 = 0.
GO TO 100,200,300,400,500,600,700,750,T
100 CALL RCCTS(C0,C1,C2,A1,A2,IER)
IF(IER.NE.0)RETURN
M1 = (C1 + A1)/C2/(A2 - A1)
M2 = -(C1 + A2)/C2/(A2 - A1)
KINV = (A2 - A1)**(M1+ M2 + 1.)*DGAMMA(M1+1.)*DGAMMA(M2+1.)
1_ /DGAMMA(M1 + M2 + 2.)
XL = A1
XR = A2
GO TO 750
200 GO TO 100
300 M1 = (C0/C1 - C1)/C1
C2 = 0.
KINV = DGAMMA(M1+1.)*DEXP(C0/C1**2)*DABS(C1)**(2.*M1+1.)
XL = -C0/C1
XR = 1.*C50
GO TO 750
400 BIGCO = C0 - C1**2/C2/4.
BIGC1 = C1/C2/2.
M1 = (C1 - BIGC1)/DSQRT(2.D01)/BIGCO
M2 = DSQRT(BIGCO/C2)
KINV = 1.D0
XL = -1.D50
XR = 1.D50
GO TO 750
500 BIGC1 = C1/2./C2

```

```

M1 = (C1 - BIGC1)/C2
IF(C2.GT.1.D0)GO TO 800
KINV = DABS(M1)**(1./C2 - 1.)/DGAMMA(1./C2-1.)
XL = -BIGC1
XR = 1.D50
GO TO 750
600 CALL ROOTS(C0,C1,C2,A1,A2,IER)
M1 = -(C1 + A1)/C2/(A2 - A1)
M2 = (C1 + A2)/C2/(A2 - A1)
IF(M2.GE.-1.D0.OR.M1 + M2.GE.0.D0)GO TO 800
KINV = (A2 - A1)**(M1 + M2 + 1.)*DGAMMA(M2+1.)*DGAMMA(-M1-M2-1.)
1 /DGAMMA(-M1)
XL = A2
XR = 1.D50
GO TO 750
700 KINV=C0**(-.5D0/C2)*DSQRT(C0/C2)*DGAMMA(.5D0)*DGAMMA(.5D0/C2-.5D0)
1 /DGAMMA(.5D0/C2)
C1 = 0.
XL = -1.D50
XR = 1.D50
750 MEAN = (4.D0*B2 - 3.D0*B1)/3.D0/(B2 - B1 - 1.D0)*U2*PDF(0.D0)
RETURN
800 IER = 1
RETURN
END

```

```

SUBROUTINE ROOTS(C0,C1,C2,A1,A2,IER)
IMPLICIT REAL*8 (A-Z)
INTEGER*4 IER
IER = 0
IF(C2.EQ.0.D0)GC TO 500
DIS = C1**2 - 4.CC*C0*C2
IF(DIS.LT.0.D0)GC TO 600
SDIS = DSQRT(DIS)
DNUM = -C1 - SDIS
IF(C1.LT.0.D0)DNUM = -C1 + SDIS
X2 = DNUM/2.D0/C2
X1 = C0/C2/X2
A1 = DMIN1(X1,X2)
A2 = DMAX1(X1,X2)
RETURN
500 A2 = 1.D75
A1 = -C0/C1
RETURN
600 IER = -1
RETURN
END

```

FUNCTION PDF(X)

THIS FUNCTION EVALUATES THE PEARSON DISTRIBUTION FOR A GIVEN X, THE PARAMETERS HAVING PREVIOUSLY BEEN CALCULATED IN SUBROUTINE PRM.

```

C
CCC
IMPLICIT REAL*8 (A-Z)
INTEGER*4 ITY
COMMON/PPARM/ C0,C1,C2,A1,A2,BIGC0,BIGC1,M1,M2,KINV,X1,X2,MEAN,I
GO TO (100,200,300,400,500,600,700),ITY
100 IF(X.LE.A1.OR.X.GE.A2)GO TO 140
PDF = (X - A1)**M1*(A2 - X)**M2/KINV
RETURN
140 IF(X.LE.A1.AND.M1.LT.0.D0)GO TO 150
IF(X.GE.A2.AND.M2.LT.0.D0)GO TO 150
PDF = 1.D-25
RETURN
150 PDF = 1.D25
RETURN
200 GO TO 100

```

```

300 PDF = (CO + C1*X)**M1*DEXP(-X/C1)/KINV
      RETURN
400 PDF = (BIGCO + C2*(X+BIGC1)**2)**(-1./C2)*
1 DEXP(-M1*DATAN((X + BIGC1)/M2))/K INV
      RETURN
500 PDF = (X + BIGC1)**(-1./C2)*DEXP(M1/(X + BIGC1))/KINV
      RETURN
600 PDF = (X - A1)**M1*(X - A2)**M2/KINV
      RETURN
700 PDF = (CC + C2*X**2)**(-.5D0/C2)/KINV
      RETURN
END

SUBROUTINE INTSZ(N,X)
IMPLICIT REAL*8 (A-Z)
COMMON /FPARM/ CO,C1,C2,A1,A2,BC0,BC1,M1,M2,KINV,XL,XR,MEAN,T
COMMON /CDF/XDUM(2001),CDF(2001),A(2001),B(2001),NP1
DIMENSION X(1)
INTEGER*4 T,N,NN,INT,IEND,NP1,IER
X(1) = A1
B(1) = PDF(A1)
CDF(1) = 0.00
INT = 1
NN = 1
DXM = (A2 - A1)/1.07
CX = DXM*1.03
GO TO 160
150 DX = DX*(1.00 + DSQRT(.475D-4/EREST))/2.
INT = 1
160 X(NN+1) = X(NN) + CX
IEND = 1
IF(X(NN+1).LT.A2)GO TO 170
X(NN+1) = A2
DX = X(NN+1) - X(NN)
IEND = 2
170 B(NN+1) = PDF(X(NN+1))
ESTINT = (B(NN+1) + B(NN))*DX/2.00
IF(ESTINT.LT.1.0E-8)GO TO 301
300 DCDF = ADINT1(X(NN),X(NN+1),ERRCR,IER)
IF(IER.GT.100)GO TO 900
IF(DCDF.GT..025D0)GC TO 309
EREST = (1.00/B(NN+1) - 1.00/B(NN))*DCDF/8.00
A(NN) = EREST
EREST = CABS(EREST)
GO TO (302,308),INT
301 DCDF = ESTINT
EREST = .2777777777D-6
A(NN) = EREST
GO TO 308
302 IF(EREST.LE..5D-4)GO TO 308
CX = DX*(DSQRT(.25D-4/EREST) + 1.00)*.5D0
IF(DX.GT.DXM)GC TO 304
DX = DXM
X(NN+1) = X(NN) + CX
INT = 2
GO TO 170
304 X(NN+1) = X(NN) + CX
IEND = 1
GO TO 170
308 CDF(NN+1) = CDF(NN) + DCDF
NN = NN + 1
IF(NN.GT.2000)GO TO 910
GO TO (150,310),IEND
309 DX = DX*.0125D0/CCDF
GO TO 160
310 N = NN - 1
NP1 = NN
DO 400 NN=1,N
400 CDF(NN+1) = CDF(NN+1)/CDF(N+1)

```

```

2 FORMAT(' CDF(NN) = ',F20.15)
RETURN
900 PRINT 1,NN,X(NN),X(NN+1),ERFCR
STOP
1 FORMAT(' I,XI,XI+1,ERFCR',I5,1P3D15.6)
910 PRINT 4;X(NN)
4 FORMAT(' RUN OUT OF SPACE AT X=',F12.5)
STOP
END

SUBROUTINE CDFINV(C,P)
REAL*8 X,CDF,A,B,CC,DDC,ADINT1,XC,E
COMMON/CDF/X(2001),CDF(2001),A(2001),B(2001),NP1
CALL TABLOC(CDF,CDFL(C),NP1,I)
XC = X(I) + (C-CDF(I))/(CDF(I+1) - CDF(I))*(X(I+1) - X(I))
IF(DABS(A(I)).LE..5D-4)GO TO 410
300 DO 400 J=1,10
DC = ADINT1(X(I),XC,E,IEF)
DDC = (CDF(I) + DC - C)/PDF(XC)
XC = XC - DDC
IF(DABS(DDC).LT..5D-4)GO TO 410
IF(XC.LE.X(I))GO TO 350
IF(XC.GE.X(I+1))GC TO 360
GO TO 400
350 XC = X(I) + (C - CDF(I))/DC*(XC - X(I))
GO TO 400
360 XC = X(I+1) + (C - CDF(I+1))/(CDF(I) + DC - CDF(I+1))*(X(I+1)-X(I))
400 CONTINUE
WRITE(6,1)C,XC
1 FORMAT(' 10 ITERATIONS IN CDFINV, C AND XC =',2E12.4)
410 P = XC
RETURN
END

```

```

SUBROUTINE TABLOC(XT,X,M,NL)
REAL*8 XT(1),X
NT = ALOG(FLOAT(M))/.301 + 1.
NU = M
NL = 1
DO 200 I=1,NT
NG = (NU + NL)/2
IF(X.GE.XT(NG))GC TC 100
NU = NG
GO TO 200
100 NL = NG
200 CONTINUE
RETURN
END

```

```

FUNCTION ADINT1(X1,X2,ERROR,IER)
C THIS SUBROUTINE USES DCADRE TO OBTAIN THE INTEGRAL OF A
CCC PEARSJN TYPE I OR II DISTRIBUTION FUNCTION. SUBTRACTING
C OUT THE SINGULARITY IS USED.
IMPLICIT REAL*8 (A-Z)
INTEGER*4 T,IER
EXTERNAL F1,F2,PDF
COMMON/PPARM/C0,C1,C2,A1,A2,B0,BC1,M1,M2,KINV,XL,XR,MEAN,T
IF(X1.GT.(XL+XR)/2.D0)GO TO 200
IF(M1.GT.0.D0)GO TO 300
PR = DCADRE(F1,X1,X2,1.D-6,0.D0,ERROR,IER)
ADINT1 = PR + (A2-A1)**M2*((X2-A1)**(M1+1.D0)-(X1-A1)**(M1+1.D0)
1 /(M1 + 1.D0)/KINV
RETURN
200 IF(M2.GT.0.D0)GO TO 300
PR = DCADRE(F2,X1,X2,1.D-6,0.D0,ERROR,IER)
ADINT1 = PR-(A2-A1)**M1*((A2-X2)**(M2+1.D0)-(A2-X1)**(M2+1.D0)

```

```

1 / (M2+1.D0)/KINV
RETURN
300 ADINT1 = DCADRE(PDF,X1,X2,1.D-6,0.D0,ERROR,IEP)
RETURN
END

FUNCTION F1(X)
IMPLICIT REAL*8 (A-Z)
COMMON /PPARM/C0,C1,C2,A1,A2,BC0,BC1,M1,M2,KINV,XL,XR,MEAN,T
IF(X.LE.A1)GO TO 200
F1 = (X-A1)**M1*((A2-X)**M2 - (A2-A1)**M2)/KINV
RETURN
200 F1 = 0.DC
RETURN
END

FUNCTION F2(X)
IMPLICIT REAL*8(A-Z)
COMMON/PPARM/C0,C1,C2,A1,A2,BC0,BC1,M1,M2,KINV,XL,XR,MEAN,T
IF(X.GE.A2)GO TO 200
F2 = (A2-X)**M2*((X-A1)**M1 - (A2-A1)**M1)/KINV
RETURN
200 F2 = 0.DC
RETURN
END

SUBROUTINE FCUTZ (PCDF,XT,NXT,FN)
THIS SUBROUTINE GENERATES THE STATISTIC FOR THE FCUTZ FN TEST.
INPUT VARIABLES ARE:
PCDF - THE CUMULATIVE DISTRIBUTION FUNCTION AGAINST WHICH THE
DEVIATES ARE BEING TESTED. CALLING SEQUENCE MUST BE OF
THE FORM 'CALL PCDF(X,P)', WHERE X IS AN INPUT VALUE,
AND THE VALUE OF THE CUMULATIVE DISTRIBUTION FUNCTION
IS RETURNED IN P.
P MUST BE BETWEEN 0 AND 1.
XT - THE ARRAY OF DEVIATES, IN INCREASING ORDER.
NXT - THE NUMBER OF DEVIATES IN THE ARRAY XT (= N - 1)

THE RETURNED VALUE IS FN, THE VALUE OF THE STATISTIC.

NXT IS PRESENTLY LIMITED TO A MAXIMUM OF 50 BY THE DIMENSION OF
THE VARIABLE XT.
DIMENSION XT(1)
REAL*8 XTD(51),RN,FND
N = NXT + 1
DO 200 I=1,NXT
K = N - I
CALL PCDF(XT(K),P)
200 XTD(K+1) = P
RN = 1.DC/N
XTD(1) = RN - XTD(2)
DO 300 I=2,NXT
300 XTD(I) = RN - XTD(I+1) + XTD(I)
XTD(N) = RN - 1.D0 + XTD(N)
FND = 0
DO 400 I=1,N
400 FND = FND + DMAX1(XTD(I),0.D0)
FN = FND
RETURN
END

```

APPENDIX 2

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10% 5% 1%	410 240 48	429 240 55	481 229 56
RSSF VS N (0,1)	10% 5% 1%	1784 1362 532	2908 2384 1410	4125 3645 2673

KOLMOGOROV-SMIRNOV TEST

		20 PTS	30 PTS	50 PTS
CCNT FOL	10% 5% 1%	491 253 49	495 253 58	483 234 46
RSSF VS N (0,1)	10% 5% 1%	1636 1008 270	2202 1401 466	3318 2410 905

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10% 5% 1%	525 225 40	481 250 35	488 253 63
RSSF VS N (0,1)	10% 5% 1%	3603 2999 1894	4010 3515 2463	4597 4279 3488

 'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFCRM(C,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.00, BETA = 0.0

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10% 5% 1%	350 220 49	421 219 57	498 242 54
RSSF VS N (0,1)	10% 5% 1%	1460 1084 401	2407 1889 1011	3616 3039 1967

KOLMOGOROV-SMIRNOV TEST

		20 PTS	30 PTS	50 PTS
CCNTROL	10% 5% 1%	488 260 64	460 244 50	500 231 38
RSSF VS N (0,1)	10% 5% 1%	1412 792 222	1782 1083 324	2654 1760 579

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTROL	10% 5% 1%	503 256 60	494 257 43	501 241 42
RSSF VS N (0,1)	10% 5% 1%	2863 2145 1102	3272 2678 1575	3963 3422 2244

 'CCNTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.30, BETA = 0.0

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	395	395	505
	5%	221	199	231
	1%	39	40	45
RSSF VS N (0,1)	10%	1441	2116	3266
	5%	1054	1598	2692
	1%	390	817	1663

KOLMOGOROV-SMIRNOV TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	493	472	502
	5%	245	215	265
	1%	50	47	52
RSSF VS N (0,1)	10%	1321	1635	2363
	5%	780	991	1508
	1%	210	294	531

FONZI F N TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	468	449	498
	5%	188	222	237
	1%	27	51	40
RSSF VS N (0,1)	10%	2479	2678	3308
	5%	1804	2059	2705
	1%	852	1080	1573

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.60, BETA = 0.0

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	389	405	491
	5%	230	221	235
	1%	43	47	48
RSSF	10%	1326	1933	3046
VS	5%	932	1469	2428
N (0,1)	1%	320	688	1440

KOLMOG-SWIR TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	527	474	489
	5%	275	236	262
	1%	59	38	54
RSSF	10%	1236	1521	2184
VS	5%	760	929	1389
N (0,1)	1%	242	271	474

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	531	505	516
	5%	265	263	262
	1%	53	49	54
RSSF	10%	2046	2274	2716
VS	5%	1396	1646	2081
N (0,1)	1%	581	788	1092

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.90, BETA = 0.0

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	1%	400 234 40	457 242 51	496 246 57
RSSF	10%	1728	2935	4056
VS	5%	1316	2435	3629

N (0,1) 1%

		555	1454	2612
--	--	-----	------	------

KOLMOG-SMIR TEST

20 PTS 30 PTS 50 PTS

CONTROL	1%	504 252 68	474 235 52	477 234 47
RSSF	10%	1565	2222	3221
VS	5%	966	1410	2372

N (0,1) 1%

		310	481	901
--	--	-----	-----	-----

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	1%	493 238 49	523 255 53	467 250 46
RSSF	10%	3502	4014	4596
VS	5%	2859	3522	4255

N (0,1) 1%

		1753	2547	3487
--	--	------	------	------

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FCRM

ALPHA = 1.00, BETA = 0.25

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10% 5% 1%	372 225 42	416 232 56	465 204 45
RSSF VS N (0,1)	10% 5% 1%	1521 1128 454	2451 1933 1057	3593 3106 2046

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10% 5% 1%	486 242 48	512 272 62	483 258 45
RSSF VS N (0,1)	10% 5% 1%	1430 890 268	1849 1171 371	2701 1882 676

FCUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10% 5% 1%	461 232 37	465 252 53	459 226 47
RSSF VS N (0,1)	10% 5% 1%	2868 2236 1174	3284 2684 1545	4026 3486 2329

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.30, EETA = 0.25

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	355	390	467
	5%	223	212	242
	1%	31	54	53

RSSF VS N (0,1)	10%	1416	2100	3277
	5%	1039	1610	2705
	1%	357	786	1607

KOLMOGOROV-SMIRNOV TEST

20 PTS 30 PTS 50 PTS

CENTFOL	10%	514	518	504
	5%	260	282	278
	1%	50	61	65

RSSF VS N (0,1)	10%	1345	1661	2402
	5%	807	1017	1618
	1%	239	331	539

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CCNTRCL	10%	493	502	495
	5%	248	260	240
	1%	45	35	53

FSSF VS N (0,1)	10%	2383	2650	3260
	5%	1698	2023	2611
	1%	822	1027	1473

 'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM
 ALPHA = 1.60, BETA = 0.25

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	402	441	497
	5%	222	230	237
	1%	43	45	48

RSSF	10%	1289	1916	3044
VS	5%	933	1458	2409
N (0,1)	1%	319	652	1328

KCLMCG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	488	469	487
	5%	234	235	242
	1%	40	52	53

RSSF	10%	1239	1471	2111
VS	5%	713	904	1357
N (0,1)	1%	223	269	445

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	484	470	494
	5%	215	246	242
	1%	56	50	49

RSSF	10%	2003	2289	2773
VS	5%	1358	1688	2098
N (0,1)	1%	552	770	1007

'CCNTRCL' IS THE TEST OF UNIFCRM(0.1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.90, BETA = 0.25

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
		10%	5%	1%
CCNTROL		383 213 45	427 248 56	499 253 37
RSSF	VS	10% 5% N (0,1)	1732 1308 539	2905 2395 1432

KCL MCG-SMIR TEST

		20 PTS	30 PTS	50 PTS
		10%	5%	1%
CCNTFCL		507 245 38	492 233 47	484 232 37
RSSF	VS	1663 1011 289	2188 1430 455	3292 2414 930
N (0,1)	1%			

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
		10%	5%	1%
CCNTROL		507 259 53	492 237 58	534 271 50
RSSF	VS	3492 2899 1822	4011 3512 2489	4571 4297 3489
N (0,1)	1%			

'CCNTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFCRM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.00, BETA = 0.50

NUMBER OF HYPOTHESES REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	382	406	442
	5%	213	195	207
	1%	45	38	38

RSSF	10%	1665	2552	3655
VS	5%	1266	2038	3222
N (0,1)	1%	509	1178	2202

KCLMCG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	501	503	486
	5%	248	256	253
	1%	45	55	57

RSSF	10%	1600	2092	3025
VS	5%	1026	1425	2208
N (0,1)	1%	342	532	951

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	506	467	471
	5%	246	236	228
	1%	44	46	43

RSSF	10%	2816	3278	3920
VS	5%	2168	2680	3382
N (0,1)	1%	1180	1546	2270

'CONTROL' IS THE TEST OF UNIFCRM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.30, BETA = 0.50

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	413	440	470
	5%	251	222	205
	1%	36	48	41

RSSF	10%	1451	2269	3435
VS	5%	1063	1794	2841
N (0,1)	1%	366	948	1826

KOLMCG-SMI R TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	478	518	513
	5%	252	278	277
	1%	49	65	60

RSSF	10%	1435	1853	2636
VS	5%	888	1229	1881
N (0,1)	1%	287	444	758

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	493	463	488
	5%	241	220	227
	1%	47	47	40

RSSF	10%	2314	2643	3268
VS	5%	1686	2028	2594
N (0,1)	1%	761	1052	1449

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.60, EETA = 0.50

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	419	411	466
	5%	218	230	234
	1%	38	54	52

RSSF	10%	1299	1934	3042
VS	5%	916	1438	2407
N (0,1)	1%	297	680	1348

KOLMOGOROV-SMIRNOV TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	502	494	483
	5%	260	255	248
	1%	47	58	63

RSSF	10%	1239	1559	2148
VS	5%	738	961	1357
N (0,1)	1%	214	296	475

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	473	479	508
	5%	259	229	240
	1%	55	53	52

RSSF	10%	2018	2310	2708
VS	5%	1384	1629	2043
N (0,1)	1%	591	749	1023

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.90, BETA = 0.50

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	410	445	486
	5%	229	236	221
	1%	30	49	46

RSSF	10%	1831	2912	4108
VS	5%	1358	1391	3659
N (0,1)	1%	566	438	2690

KOLMOG-SMIR T TEST

20 PTS 30 PTS 50 PTS

CCNTROL	10%	524	505	527
	5%	250	259	251
	1%	46	53	55

RSSF	10%	1627	2196	3304
VS	5%	1051	1421	2387
N (0,1)	1%	304	500	931

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	515	497	502
	5%	229	254	255
	1%	51	59	46

RSSF	10%	3586	4057	4542
VS	5%	3005	3591	4256
N (0,1)	1%	1893	2495	3453

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.00, EETA = C.75

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	368	447	505
	5%	200	241	223
	1%	43	58	39

RSSF VS N (0,1)	10%	1941	2611	4097
	5%	1495	2429	3670
	1%	704	1535	2762

KOLMOGOROV-SMIRNOV TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	475	504	524
	5%	242	237	264
	1%	47	45	47

RSSF VS N (0,1)	10%	2017	2640	3736
	5%	1331	1930	3089
	1%	546	904	1762

F CUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	461	513	490
	5%	226	248	248
	1%	45	54	55

RSSF VS N (0,1)	10%	2895	3312	4051
	5%	2237	2712	3542
	1%	1209	1595	2426

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFCRM(C,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.30, BETA = 0.75

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	400	407	501
	5%	243	225	256
	1%	48	51	46

RSSF	10%	1628	2451	3633
VS	5%	1240	1965	3099
N (0,1)	1%	509	1126	2110

KOLMOG-SMIR TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	491	509	476
	5%	227	244	243
	1%	48	54	50

RSSF	10%	1622	2104	3023
VS	5%	1050	1465	2330
N (0,1)	1%	379	588	1145

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	522	490	450
	5%	266	267	243
	1%	52	60	56

RSSF	10%	2426	2722	3367
VS	5%	1753	2067	2727
N (0,1)	1%	825	1058	1614

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.60, BETA = 0.75

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10% 5% 1%	405 238 54	407 229 49	465 215 46
RSSF VS N (0,1)	10% 5% 1%	1366 987 341	1989 1490 704	3088 2487 1419

KOLMOGOROV-SMIRNOV TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10% 5% 1%	501 246 57	505 268 53	483 245 46
RSSF VS N (0,1)	10% 5% 1%	1273 797 234	1558 936 308	2246 1465 508

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10% 5% 1%	497 242 58	457 227 48	478 242 48
RSSF VS N (0,1)	10% 5% 1%	2102 1479 647	2256 1615 760	2745 2081 1076

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FCRM
ALPHA = 1.90, BETA = 0.75

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	395	441	495
	5%	213	238	243
	1%	35	53	62

RSSF	10%	1793	2920	4109
VS	5%	1395	2386	3658
N (0,1)	1%	554	1413	2707

KOLMOG-SMIR TEST

20 PTS 30 PTS 50 PTS

CONTCL	10%	493	504	457
	5%	241	231	238
	1%	50	43	43

RSSF	10%	1640	2203	3325
VS	5%	1004	1392	2434
N (0,1)	1%	295	421	939

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTCL	10%	526	488	481
	5%	260	247	223
	1%	54	58	50

RSSF	10%	3530	4001	4577
VS	5%	2933	3501	4284
N (0,1)	1%	1849	2405	3405

'CONTCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.00, BETA = 1.00

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	363	428	518
	5%	218	226	242
	1%	45	44	55

RSSF VS N (0,1)	10%	2651	3826	4682
	5%	2291	3457	4472
	1%	1344	2572	3935

KOLMOGOROV-SMIRNOV TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	526	533	524
	5%	243	268	282
	1%	51	48	70

RSSF VS N (0,1)	10%	2937	3776	4638
	5%	2326	3235	4378
	1%	1245	2035	3452

FOLKERTZ-FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	488	477	500
	5%	259	216	238
	1%	54	53	44
RSSF VS N (0,1)	10%	2962	3429	4162
	5%	2260	2813	3660
	1%	1237	1705	2550

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.30, EETA = 1.00

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CCNT FOL	10%	412	443	472
	5%	230	237	220
	1%	30	50	46

RSSF	10%	1868	2768	3558
VS	5%	1487	2294	3541
N (C,1)	1%	701	1427	2578

KOLMOGOROV-SMIRNOV TEST

20 PTS 30 PTS 50 PTS

CCNTROL	10%	513	477	514
	5%	268	251	251
	1%	56	56	49

RSSF	10%	1968	2603	3652
VS	5%	1370	1903	3016
N (0,1)	1%	586	905	1765

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	507	504	505
	5%	250	240	252
	1%	58	45	58

RSSF	10%	2414	2792	3409
VS	5%	1786	2120	2771
N (C,1)	1%	853	1123	1625

'CCNTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.60, BETA = 1.00

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	374	431	525
	5%	231	232	263
	1%	48	57	58

RSSF	10%	1339	1553	3107
VS	5%	977	1438	2466
N (0,1)	1%	347	708	1459

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTROL	10%	494	481	516
	5%	239	240	241
	1%	54	47	43

RSSF	10%	1322	1625	2308
VS	5%	808	978	1533
N (0,1)	1%	235	302	540

FOUTZ FN T TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	525	523	520
	5%	236	271	296
	1%	50	67	55

RSSF	10%	2086	2280	2758
VS	5%	1409	1645	2075
N (0,1)	1%	620	715	1078

 'CCNTFCL' IS THE TEST OF UNIFORM(0,1) VS UNIFCRM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.90, BETA = 1.00

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10% 5% 1%	388 229 39	469 238 48	486 224 43
MIXNCRM	10% 5% 1%	1272 909 329	1925 1407 664	3004 2381 1337
N (0,1)				

KOLMOGOROV-SMIRNOV TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10% 5% 1%	529 254 42	513 260 46	510 249 64
MIXNCRM	10% 5% 1%	1235 713 208	1537 931 285	2133 1369 449
N (0,1)				

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10% 5% 1%	508 245 50	476 222 38	523 255 60
MIXNCRM	10% 5% 1%	1982 1336 533	2191 1576 672	2610 1995 977
N (0,1)				

 'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

C. C *N(.0,1) + 1.00*N(C,2)

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	409	411	498
	5%	233	205	234
	1%	29	34	46
MIXNCRM	10%	2513	3558	4620
VS	5%	2088	3080	4342
N (0,1)	1%	1028	2052	3609

KOLMOGOROV-SMIRNOV TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	475	463	500
	5%	238	229	242
	1%	46	35	38
MIXNCRM	10%	2188	2827	3965
VS	5%	1420	1974	3161
N (0,1)	1%	512	727	1515

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	522	454	481
	5%	236	217	226
	1%	47	41	52
MIXNCRM	10%	3338	3712	4346
VS	5%	2724	3174	3935
N (0,1)	1%	1657	2064	2970

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

C.C *N(.0,1) + 1.00*N(0,3)

NUMBER OF HYPOTHESES REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
	10%	407	425	511
CONTROL	5%	234	232	252
	1%	48	48	50

MIXNORM	10%	3427	4423	4935
VS	5%	3006	4154	4863
N (0,1)	1%	1781	3377	4639

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
	10%	510	490	511
CCNTROL	5%	261	241	250
	1%	59	48	47

MIXNCRM	10%	2976	3799	4717
VS	5%	2113	3025	4286
N (0,1)	1%	890	1471	2792

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
	10%	518	501	528
CCNTFOL	5%	242	268	276
	1%	68	66	62
MIXNCRM	10%	4131	4502	4865
VS	5%	3686	4195	4742
N (0,1)	1%	2653	3380	4272

 CONTROL IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

MIXNCRM IS THE MIXED NCRMAL DISTRIBUTION

0.0 *N(.0,1) + 1.00*N(0,4)

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CCNTFOL	10%	382	422	510
	5%	215	187	254
	1%	41	38	46

MIXNCRM	10%	472	542	713
VS	5%	293	308	370
N (0,1)	1%	29	66	74

KOLMOG-SMIR TEST

20 PTS 30 PTS 50 PTS

CCNTFCL	10%	488	537	525
	5%	238	250	265
	1%	46	50	53

MIXNCRM	10%	587	667	695
VS	5%	287	328	367
N (0,1)	1%	54	70	84

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CCNTFCL	10%	468	480	517
	5%	220	258	247
	1%	46	64	56

MIXNCRM	10%	799	796	881
VS	5%	429	426	477
N (0,1)	1%	105	107	136

'CCNTFCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'MIXNORM' IS THE MIXED NORMAL DISTRIBUTION

C.70*N(.0,1) + C.30*N(0,2)

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CCNT FOL	10%	421	457	510
	5%	245	244	221
	1%	49	52	38

MIXNCRM	10%	551	727	1047
VS	5%	345	438	602
N (0,1)	1%	82	131	185

KOLMOOG-SMIR TEST

20 PTS 30 PTS 50 PTS

CCNTROL	10%	490	504	527
	5%	251	256	239
	1%	46	58	42

MIXNORM	10%	646	745	845
VS	5%	337	405	447
N (0,1)	1%	81	93	83

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	519	494	507
	5%	240	242	256
	1%	55	46	51

MIXNCFM	10%	1029	1139	1249
VS	5%	591	671	778
N (0,1)	1%	172	193	262

'CCNTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'MIXNORM' IS THE MIXED NORMAL DISTRIBUTION

0.7C*N(.0,1) + 0.30*N(0,3)

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTROL	10%	400	435	515
	5%	237	228	240
	1%	45	50	52
MIXNCRM	10%	636	510	1320
VS	5%	405	564	836
N (0,1)	1%	109	175	322

KOLMOGOROV-SMIRNOV TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	508	531	508
	5%	259	259	248
	1%	54	51	54
MIXNCRM	10%	747	853	1001
VS	5%	407	441	532
N (0,1)	1%	115	122	148

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	492	478	511
	5%	229	232	241
	1%	61	47	43
MIXNCRM	10%	1281	1418	1666
VS	5%	774	888	1121
N (0,1)	1%	238	322	401

 'CCNTFCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNORM' IS THE MIXED NORMAL DISTRIBUTION

0.70*N(.0,1) + 0.30*N(0,4)

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	380	431	507
	5%	221	245	220
	1%	40	59	56

MIXNCRM	10%	388	484	545
VS	5%	214	274	275
N (0,1)	1%	40	63	62

KOLMOGOROV-SMIRNOV TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	468	480	481
	5%	220	236	232
	1%	53	46	38

MIXNCRM	10%	519	539	556
VS	5%	258	277	286
N (0,1)	1%	58	55	53

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	487	498	502
	5%	234	242	265
	1%	42	49	54

MIXNCFM	10%	684	689	687
VS	5%	352	354	400
N (0,1)	1%	83	92	96

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

C.8C*N(.0,1) + 0.20*N(0,2)

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	417	405	492
	5%	218	211	243
	1%	43	38	42

MIXNCRM	10%	477	544	736
VS	5%	281	306	364
N (0,1)	1%	48	64	98

KOLMOG-SMIR TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	500	469	537
	5%	267	222	260
	1%	47	46	48

MIXNCRM	10%	587	590	678
VS	5%	317	297	342
N (0,1)	1%	69	59	78

FCUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	495	462	479
	5%	234	213	245
	1%	49	44	48

MIXNCRM	10%	839	819	945
VS	5%	475	454	522
N (0,1)	1%	118	130	143

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

0.8C*N(0,1) + 0.20*N(0,3)

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	402	450	539
	5%	225	242	247
	1%	43	49	60
MIXNORM	10%	507	637	902
VS	5%	310	361	456
N (0,1)	1%	74	89	138

KOLMOGOROV-SMIRNOV TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	511	502	543
	5%	253	252	270
	1%	51	43	63
MIXNORM	10%	622	661	773
VS	5%	329	360	416
N (0,1)	1%	85	69	53

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	485	496	514
	5%	238	243	255
	1%	48	48	55
MIXNORM	10%	958	1011	1168
VS	5%	548	583	721
N (0,1)	1%	171	174	246

 'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNORM' IS THE MIXED NORMAL DISTRIBUTION
 $0.80*N(0,1) + 0.20*N(0,4)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10% 5% 1%	420 231 40	470 240 51	470 238 44
MIXNORM VS N (0,1)	10% 5% 1%	410 228 35	478 241 48	525 247 46

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10% 5% 1%	499 244 54	490 245 44	519 251 55
MIXNCRM VS N (0,1)	10% 5% 1%	531 264 57	512 257 47	535 266 64

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10% 5% 1%	493 245 43	490 256 62	480 229 47
MIXNCRM VS N (0,1)	10% 5% 1%	577 282 66	583 310 73	587 290 77

 'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNORM' IS THE MIXED NORMAL DISTRIBUTION

$$0.90*N(.0,1) + 0.10*N(0,2)$$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIUNS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	432	416	486
	5%	246	214	215
	1%	43	41	40

MIXNCRM	10%	456	431	537
VS	5%	262	233	258
N (0,1)	1%	51	43	53

KOLMOG-SMIR TEST

20 PTS 30 PTS 50 PTS

CONT FOL	10%	509	495	505
	5%	271	239	252
	1%	55	46	55

MIXNCRM	10%	553	526	557
VS	5%	302	261	274
N (0,1)	1%	67	53	64

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	516	471	520
	5%	271	239	252
	1%	58	51	47

MIXNCRM	10%	660	653	675
VS	5%	353	357	356
N (0,1)	1%	95	88	89

 'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFCRM(0,1)

'MIXNCRM' IS THE MIXED NCRMAL DISTRIBUTION

C.5C*N(.0,1) + 0.1C*N(0,3)

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	397	439	509
	5%	240	244	246
	1%	31	53	55
MIXNCRM	10%	419	508	551
VS	5%	260	266	302
N (0,1)	1%	36	65	74

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTFOL	10%	484	516	488
	5%	240	247	239
	1%	56	47	46
MIXNCRM	10%	525	584	548
VS	5%	268	282	277
N (0,1)	1%	64	61	60

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTFOL	10%	514	511	491
	5%	247	253	245
	1%	48	50	58
MIXNCRM	10%	736	707	747
VS	5%	385	384	422
N (0,1)	1%	81	105	112

 'CCNTFOL' IS THE TEST OF UNIFORM(0,1) VS UNIFCRM(0,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

C.90*N(.0,1) + 0.10*N(0,4)

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CCNTRCL	10%	386	417	454
	5%	225	241	238
	1%	39	44	46

MIXNCRM	10%	1459	2008	3146
VS	5%	1093	1533	2556
N (C,1)	1%	407	753	1568

KOLMOGOROV-SMIRNOV TEST

20 PTS 30 PTS 50 PTS

CCNTROL	10%	493	483	487
	5%	267	246	238
	1%	52	53	38

MIXNCRM	10%	2179	2806	3756
VS	5%	1606	2181	3226
N (C,1)	1%	714	1148	2043

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	535	495	499
	5%	250	246	248
	1%	54	43	41

MIXNORM	10%	1809	1994	2492
VS	5%	1198	1428	1825
N (0,1)	1%	509	602	885

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM (0,1)

'MIXNORM' IS THE MIXED NORMAL DISTRIBUTION

C.70*N(.5,1) + 0.30*N(C,3)

NUMBER OF HYPOTHESES REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	377	449	512
	5%	205	239	257
	1%	46	46	44

MIXNCRM	10%	1557	2112	3304
VS	5%	1170	1563	2660
N (0,1)	1%	473	750	1640

KOLMCG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTROL	10%	482	481	538
	5%	229	260	271
	1%	45	69	56

MIXNCRM	10%	2450	3101	4056
VS	5%	1855	2510	3596
N (0,1)	1%	901	1344	2400

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTROL	10%	479	512	531
	5%	233	262	255
	1%	50	53	53
MIXNCRM	10%	1781	1917	2405
VS	5%	1209	1369	1774
N (0,1)	1%	462	573	843

 'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

$$0.80*N(.5,1) + 0.20*N(0,3)$$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

	20 PTS	30 PTS	50 PTS
--	--------	--------	--------

CONTROL	10% 5% 1%	389 221 38	439 251 55	490 221 49
MIXNCRM	10% 5% N (0,1)	1568 1177 507	2228 1736 951	3377 2832 1796

KOLMOGOROV-SMIRNOV TEST

	20 PTS	30 PTS	50 PTS
--	--------	--------	--------

CONTROL	10% 5% 1%	535 258 55	489 253 54	490 245 35
MIXNCRM	10% 5% N (0,1)	2467 1865 857	3194 2593 1438	4105 3659 2511

F CUTZ FN TEST

	20 PTS	30 PTS	50 PTS
--	--------	--------	--------

CONTROL	10% 5% 1%	502 252 62	496 250 51	495 235 53
MIXNCRM	10% 5% N (0,1)	1838 1220 486	2156 1551 701	2665 1978 979

 "CONTROL" IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

"MIXNCRM" IS THE MIXED NORMAL DISTRIBUTION

$$0.8C^*N(.5, 1) + 0.20*N(0, 4)$$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTROL	10%	431	413	528
	5%	240	200	249
	1%	40	42	53

PEARSON	10%	465	478	626
VS	5%	258	242	302
N (0,1)	1%	46	59	71

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTROL	10%	528	478	525
	5%	275	242	283
	1%	45	49	54

PEARSON	10%	623	571	673
VS	5%	339	310	371
N (0,1)	1%	66	70	95

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTROL	10%	512	458	530
	5%	248	207	262
	1%	56	48	58
PEARSON	10%	481	428	497
VS	5%	241	196	234
N (0,1)	1%	60	36	57

'CCNTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 0.0 , BETA 2 = 2.30

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	435	443	541
	5%	237	241	247
	1%	49	52	49

PEARSON	10%	432	445	533
VS	5%	243	247	241
N (0,1)	1%	43	55	52

KOLMOGOROV-SMIRNOV TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	489	528	489
	5%	238	252	245
	1%	55	50	64

PEARSON	10%	496	543	503
VS	5%	242	274	262
N (0,1)	1%	61	54	67

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	536	467	487
	5%	275	227	240
	1%	61	34	43

PEARSON	10%	523	441	467
VS	5%	269	217	228
N (0,1)	1%	58	33	43

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 0.0 , BETA 2 = 2.80

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CCNTROL	10%	408	390	453
	5%	230	214	213
	1%	35	50	52
PEARSCN	10%	756	992	1576
VS	5%	495	634	973
N (0,1)	1%	121	185	343

KOLMOGOROV-SMIRNOV TEST

20 PTS 30 PTS 50 PTS

CCNTROL	10%	492	515	520
	5%	252	255	264
	1%	55	49	52
PEARSCN	10%	942	1161	1498
VS	5%	546	693	944
N (0,1)	1%	158	192	294

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CCNTROL	10%	505	478	504
	5%	223	261	255
	1%	50	56	48
PEARSCN	10%	853	1400	2487
VS	5%	469	884	1785
N (0,1)	1%	129	311	799

'CCNTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'PEARSCN' IS THE PEARSON DISTRIBUTION

BETA 1 = 0.01, BETA 2 = 1.75

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	428	410	493
	5%	239	205	233
	1%	40	43	52

PEARSON	10%	614	744	1083
VS	5%	375	435	610
N (C,1)	1%	93	120	193

KOLMOGOROV-SMIRNOV TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	543	470	506
	5%	290	229	274
	1%	41	50	48

PEARSON	10%	837	857	1106
VS	5%	482	479	648
N (C,1)	1%	131	131	184

FONZI FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	511	474	515
	5%	240	214	259
	1%	55	49	54

PEARSON	10%	558	726	1288
VS	5%	305	405	774
N (C,1)	1%	82	101	233

'CONTROL' IS THE TEST OF UNIFORM(C,1) VS UNIFORM(0,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 0.01, BETA 2 = 1.90

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10% 5% 1%	408 230 48	442 237 41	510 241 47
PEARSON	10% 5% 1% (C,1)	458 267 51	556 321 74	702 366 56

KOLMOGOROV-SMIRNOV TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10% 5% 1%	492 240 55	485 272 50	502 258 65
PEARSON	10% 5% 1% (C,1)	581 309 74	650 329 73	715 389 94

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10% 5% 1%	529 241 62	479 241 56	546 285 54
PEARSON	10% 5% 1% (C,1)	534 269 68	554 283 66	666 353 81

 'CCNTFCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 0.25, BETA 2 = 3.20

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	373	419	502
	5%	220	230	211
	1%	55	49	34

PEARSON	10%	739	1030	1672
VS	5%	476	664	1099
N (0,1)	1%	118	231	431

KOLMOGOROV-SMIRNOV TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	516	483	475
	5%	253	227	231
	1%	48	51	38

PEARSON	10%	790	896	1125
VS	5%	474	528	735
N (0,1)	1%	121	147	234

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	499	481	477
	5%	237	238	253
	1%	46	49	50

PEARSON	10%	868	1090	1518
VS	5%	471	660	965
N (0,1)	1%	121	185	344

 'CONTROL' IS THE TEST OF UNIFORM(0,1). VS UNIFORM(C,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 0.50, BETA 2 = 3.00

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CENTFCL	10%	401	428	522
	5%	236	225	239
	1%	43	39	54

FEARSCN	10%	2440	3386	4654
VS	5%	1960	2820	4320
N (C,1)	1%	931	1632	3267

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CENTFCL	10%	482	513	505
	5%	239	254	245
	1%	47	52	61

FEARSCN	10%	1155	1459	2001
VS	5%	730	920	1428
N (C,1)	1%	216	322	609

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CENTFCL	10%	531	492	550
	5%	250	250	277
	1%	57	51	61

FEARSCN	10%	2135	2837	3920
VS	5%	1460	2156	3309
N (C,1)	1%	583	1049	2104

 'CENTFCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'FEARSCN' IS THE PEARSON DISTRIBUTION

BETA 1 = 1.00, BETA 2 = 3.40

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CCNTFCL	10%	411	427	475
	5%	226	238	227
	1%	34	49	34
PEARSON	10%	1446	3151	4513
VS	5%	1070	2546	4039
N (C,1)	1%	403	1368	2895

KOLMOGOROV-SMIRNOV TEST

20 PTS 30 PTS 50 PTS

CCNTFCL	10%	499	495	504
	5%	249	260	269
	1%	55	64	60
PEARSON	10%	1032	1314	1765
VS	5%	649	832	1227
N (C,1)	1%	211	265	453

F C U T Z F N TEST

20 PTS 30 PTS 50 PTS

CCNTFCL	10%	482	481	485
	5%	251	251	207
	1%	51	45	43
PEARSON	10%	1578	2161	3119
VS	5%	1009	1493	2354
N (C,1)	1%	348	633	1254

'CCNTFCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 1.00, BETA 2 = 3.60

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	397	427	485
	5%	232	233	234
	1%	41	60	56

PEARSCN	10%	1086	2009	3236
VS	5%	764	1463	2543
N (0,1)	1%	255	651	1382

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	524	501	459
	5%	267	250	252
	1%	51	56	62

PEARSCN	10%	911	1150	1528
VS	5%	546	732	1032
N (0,1)	1%	171	229	399

FOOTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	521	460	529
	5%	258	248	260
	1%	50	63	46

PEARSCN	10%	1261	1656	2435
VS	5%	764	1070	1706
N (0,1)	1%	243	387	739

 'CCNTFCL' IS THE TEST OF UNIFCRM(0,1) VS UNIFORM(0,1)

'PEARSCN' IS THE PEARSON DISTRIBUTION

BETA 1 = 1.00, BETA 2 = 3.80

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	371	431	464
	5%	206	237	227
	1%	37	48	45

FEARSCN	10%	2231	3039	4476
VS	5%	1745	2383	3938
N (0,1)	1%	763	1242	2652

KOLMOGOROV-SMIRNOV TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	498	482	517
	5%	250	261	261
	1%	42	48	54

FEARSCN	10%	1153	1481	2082
VS	5%	753	1005	1447
N (0,1)	1%	231	353	621

FOUTZ FN TEST

20 PTS 30 PTS 50 PTS

CONTROL	10%	450	498	486
	5%	258	255	248
	1%	44	45	61

FEARSCN	10%	1872	2587	3548
VS	5%	1230	1920	2894
N (0,1)	1%	469	886	1644

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'FEARSCN' IS THE PEARSON DISTRIBUTION

BETA 1 = 2.0C, BETA 2 = 5.50

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

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